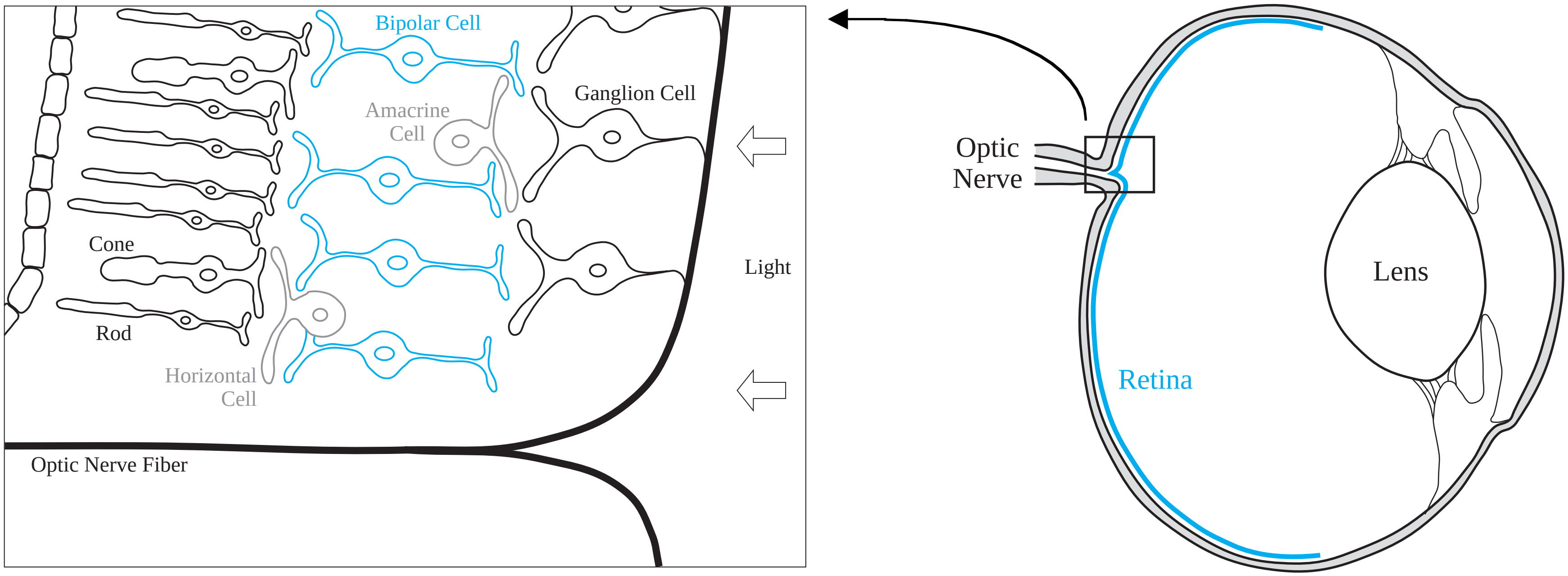
Grossberg Network

Biological Motivation: Vision



Eyeball and Retina

Layers of Retina

The retina is a part of the brain that covers the back inner wall of the eye and consists of three layers of neurons:

Outer Layer:
- Photoreceptors - convert light into electrical signals
  - Rods - allow us to see in dim light
  - Cones - fine detail and color

Middle Layer
- Bipolar Cells - link photoreceptors to third layer
- Horizontal Cells - link receptors with bipolar cells
- Amacrine Cells - link bipolar cells with ganglion cells

Final Layer
- Ganglion Cells - link retina to brain through optic nerve

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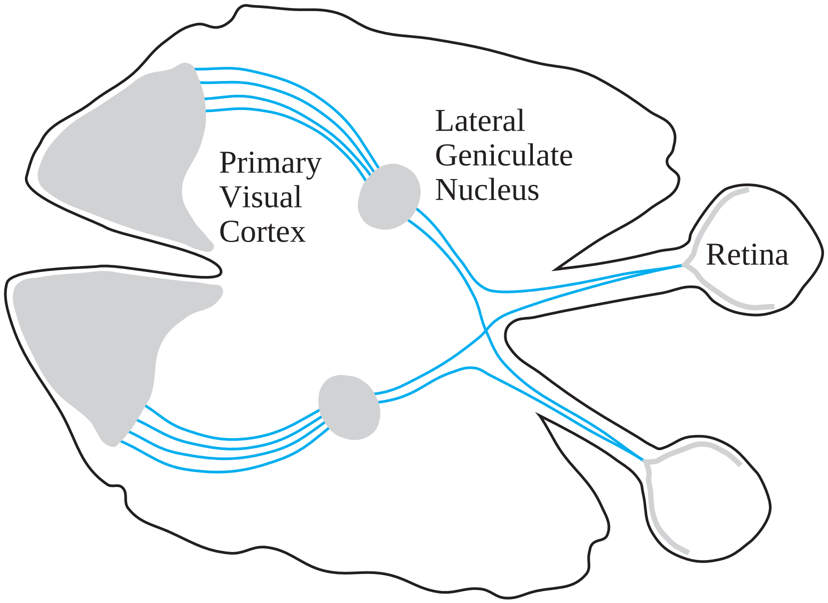
Visual Pathway



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Photograph of the Retina

Blind Spot (Optic Disk)

Vein

Fovea

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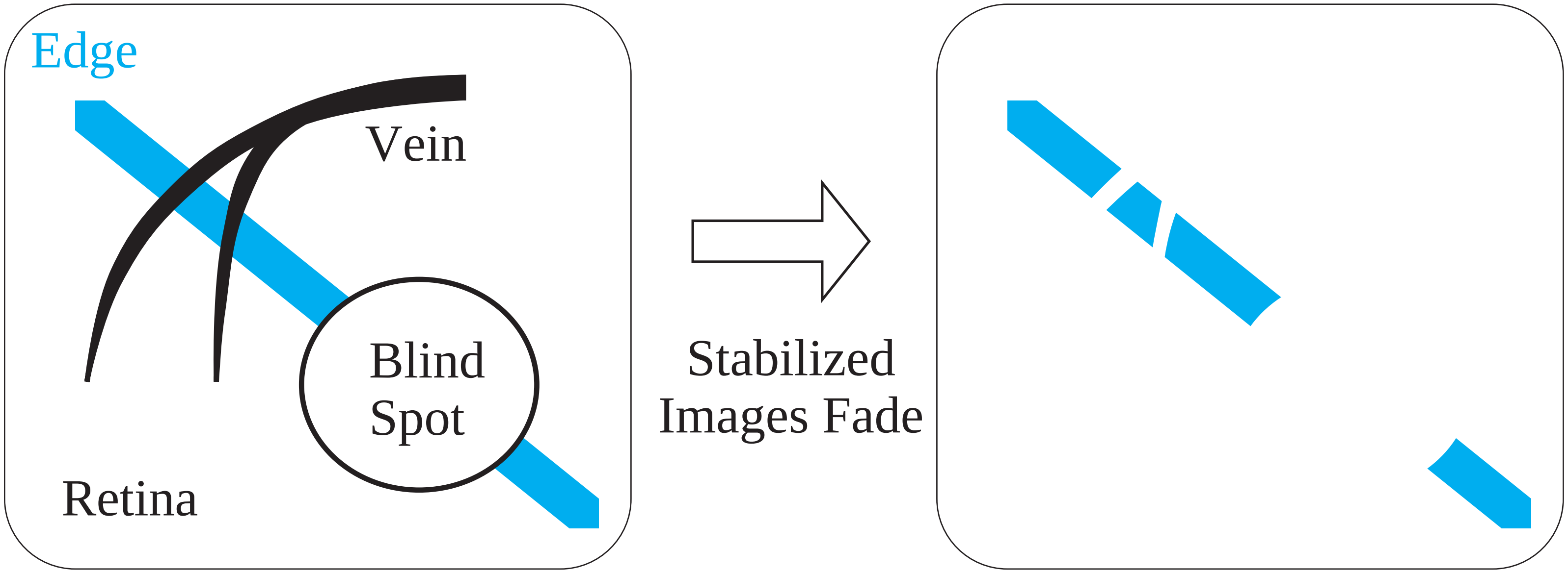
Imperfections in Retinal Uptake



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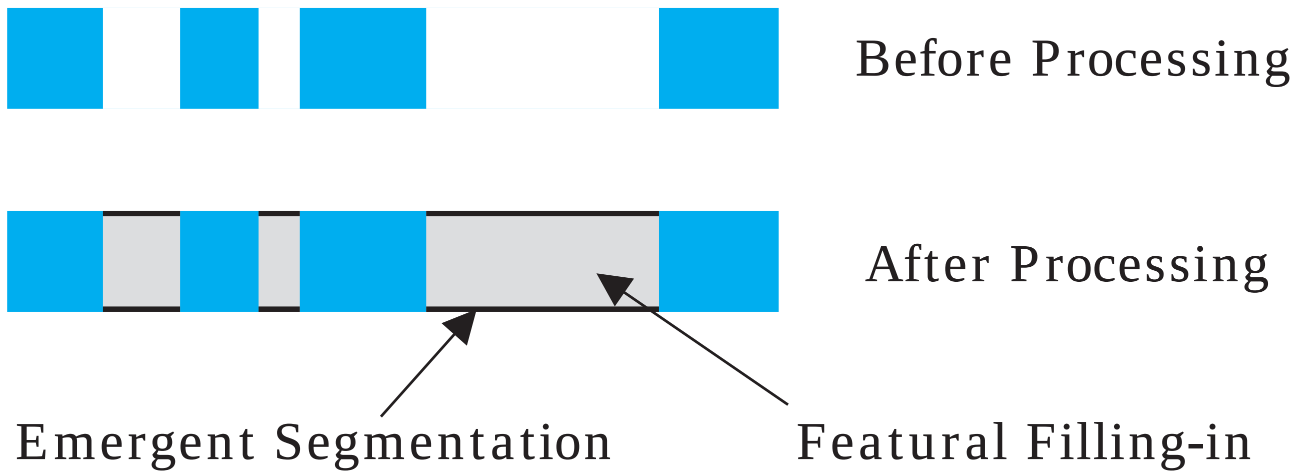
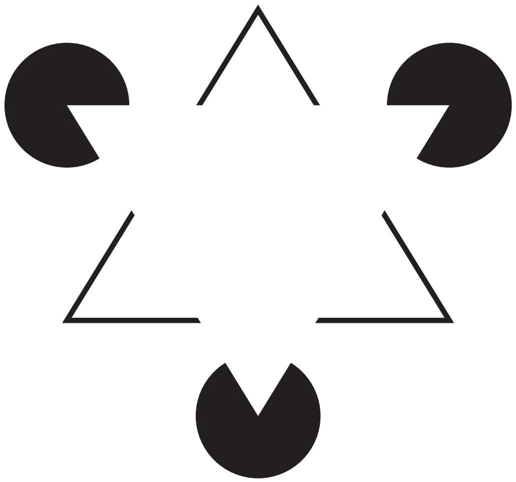
Compensatory Processing

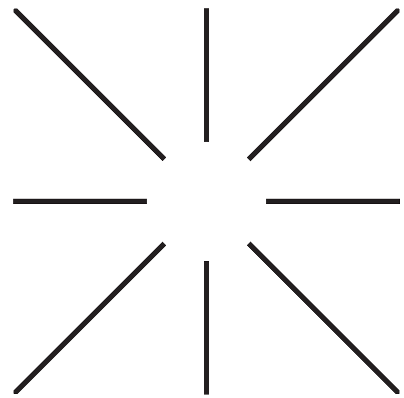
Emergent Segmentation:
Complete missing boundaries.

Featural Filling-In:
Fill in color and brightness.



Visual Illusions





Illusions demostrate the compensatory processing of the visual system. Here we see a bright white triangle and a circle which do not actually exist in the figures.

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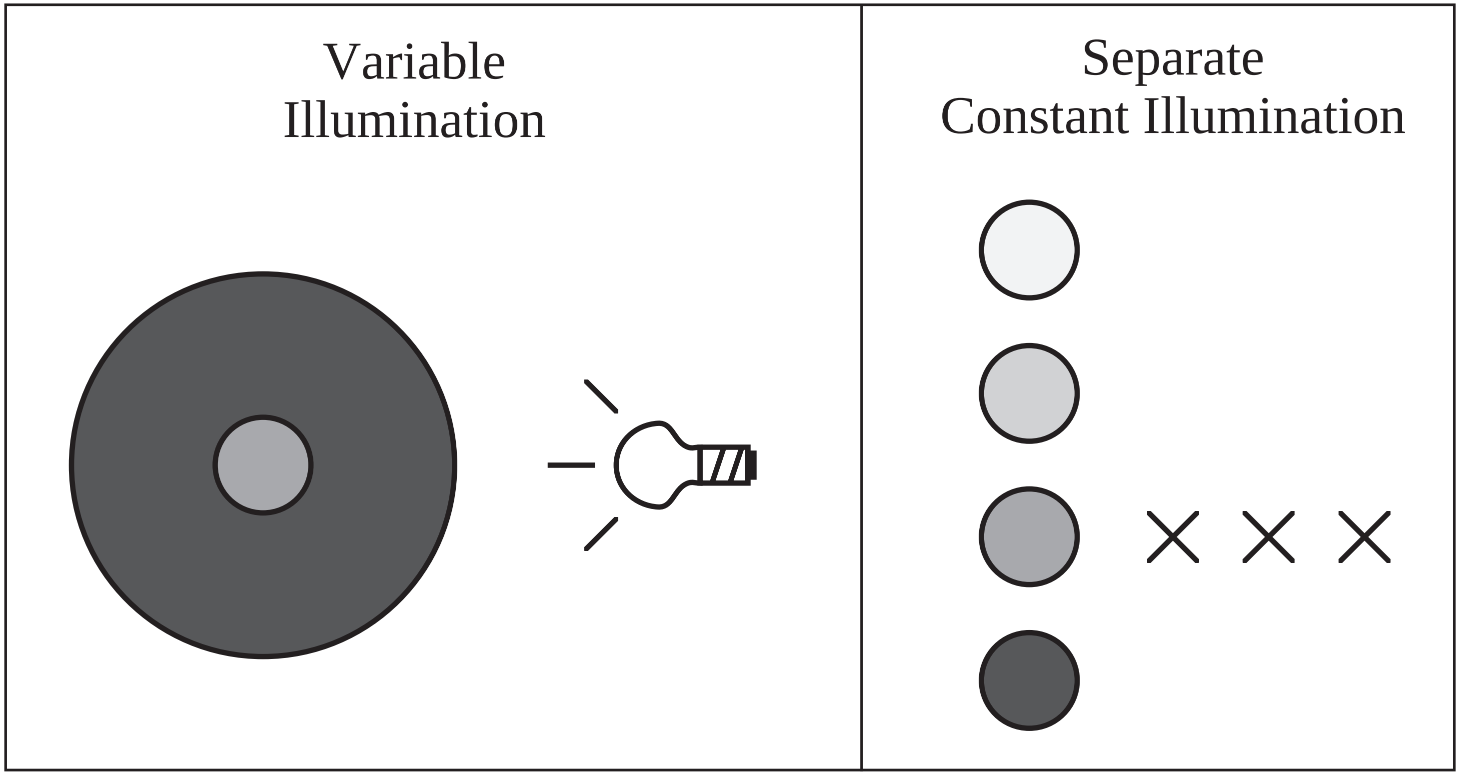
Vision Normalization



The vision systems normalize scenes so that we are only aware of relative differences in brightness, not absolute brightness.

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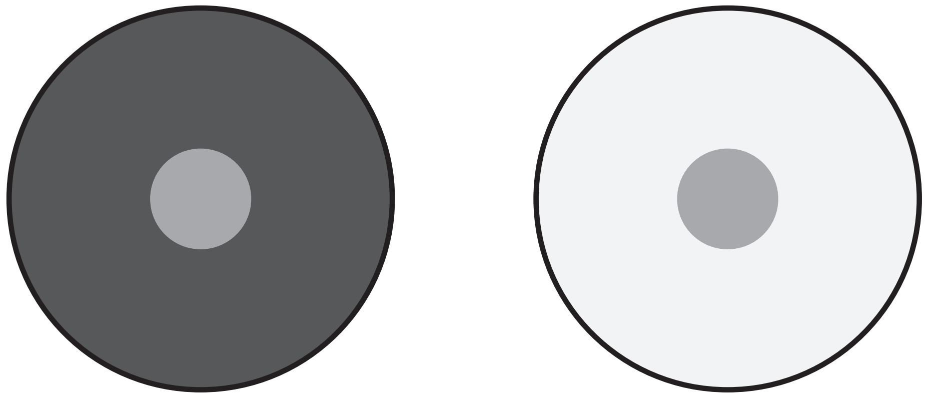
Brightness Contrast



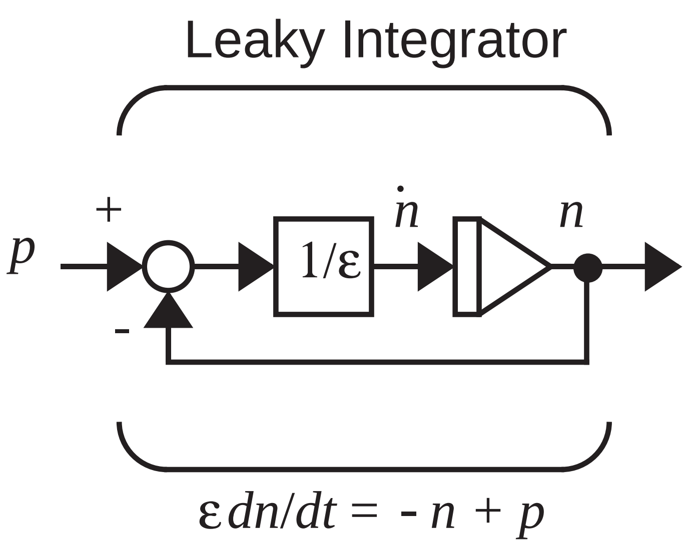
If you look at a point between the two circles, the small inner circle on the left will appear lighter than the small inner circle on the right, although they have the same brightness. It is relatively lighter than its surroundings.

The visual system normalizes the scene. We see relative intensities.

Leaky Integrator

(Building block for basic nonlinear model.)

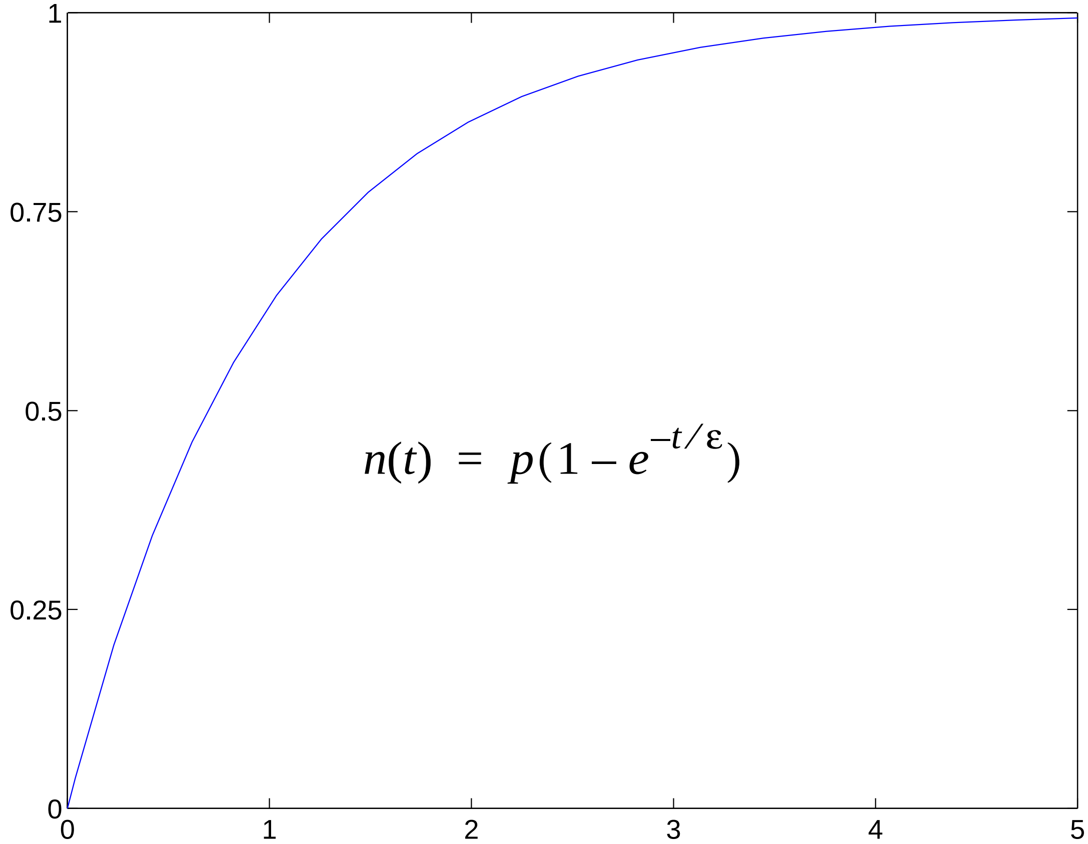
$$\varepsilon\frac{dn(t)}{dt} = -n(t) + p(t)$$



15 Leaky Integrator Response

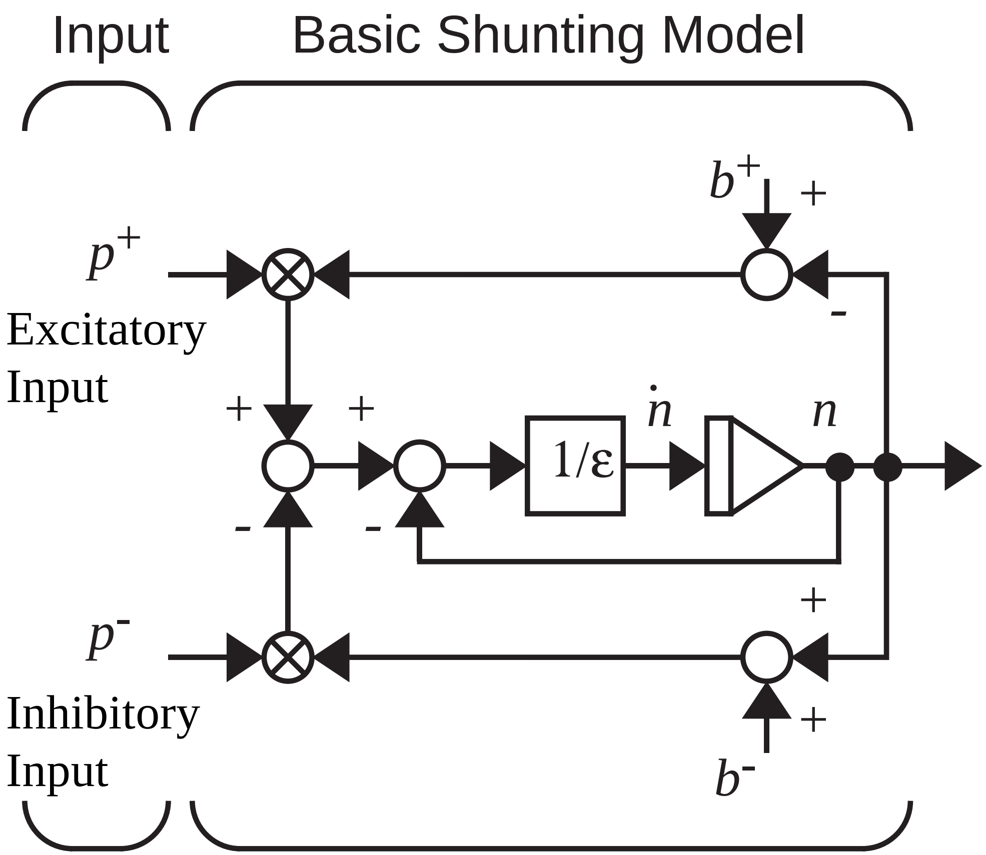
$$n(t) = e^{-t/\varepsilon} n(0) + \frac{1}{\varepsilon}\int_0^t e^{-(t-\tau)/\varepsilon} p(t-\tau)\, d\tau$$

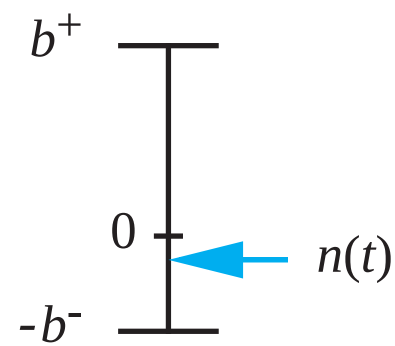
For a constant input and zero initial conditions:



$$n(t) = p(1 - e^{-t/\varepsilon})$$

Shunting Model





$$\varepsilon\, dn/dt = -n + \underbrace{(b^{+} - n)}_{\text{}}p^{+} - \underbrace{(n + b^{-})}_{\text{}}p^{-}$$

Gain Control (Sets upper limit)

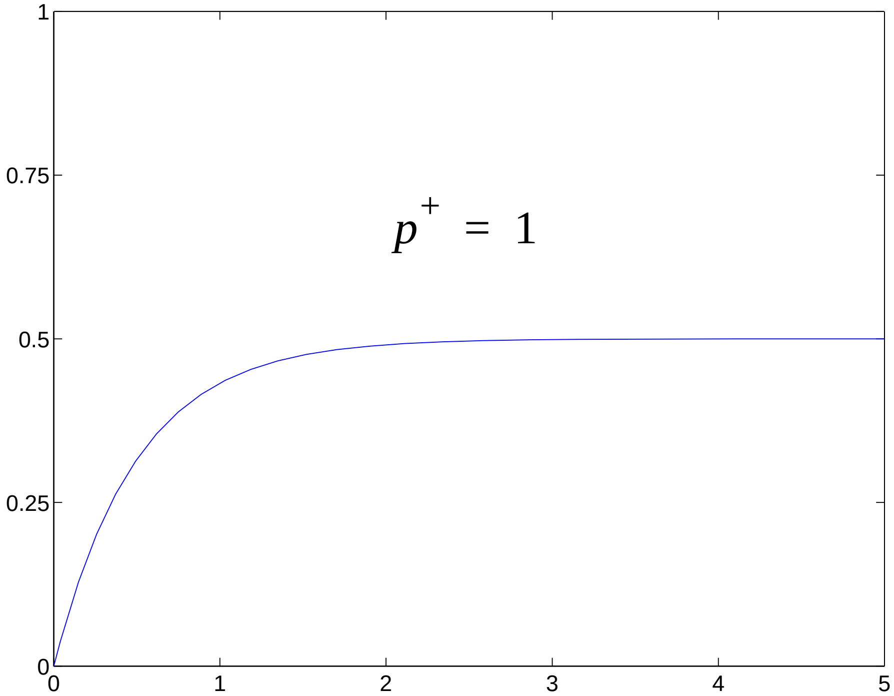
Gain Control (Sets lower limit)

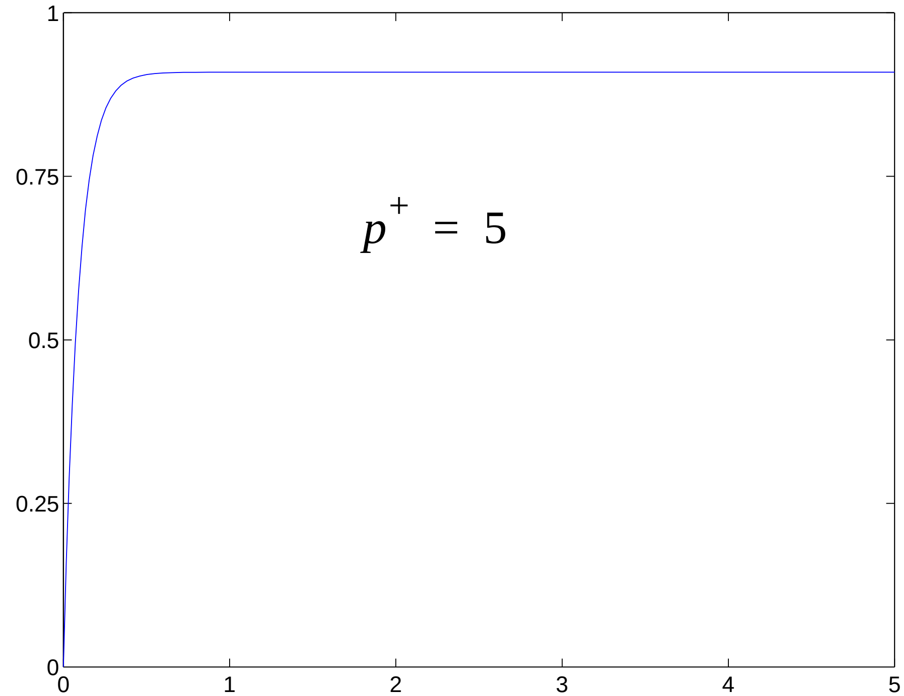
15 Shunting Model Response

$$\varepsilon \frac{dn(t)}{dt} = -n(t) + (b^{+} - n(t))p^{+} - (n(t) + b^{-})p^{-}$$

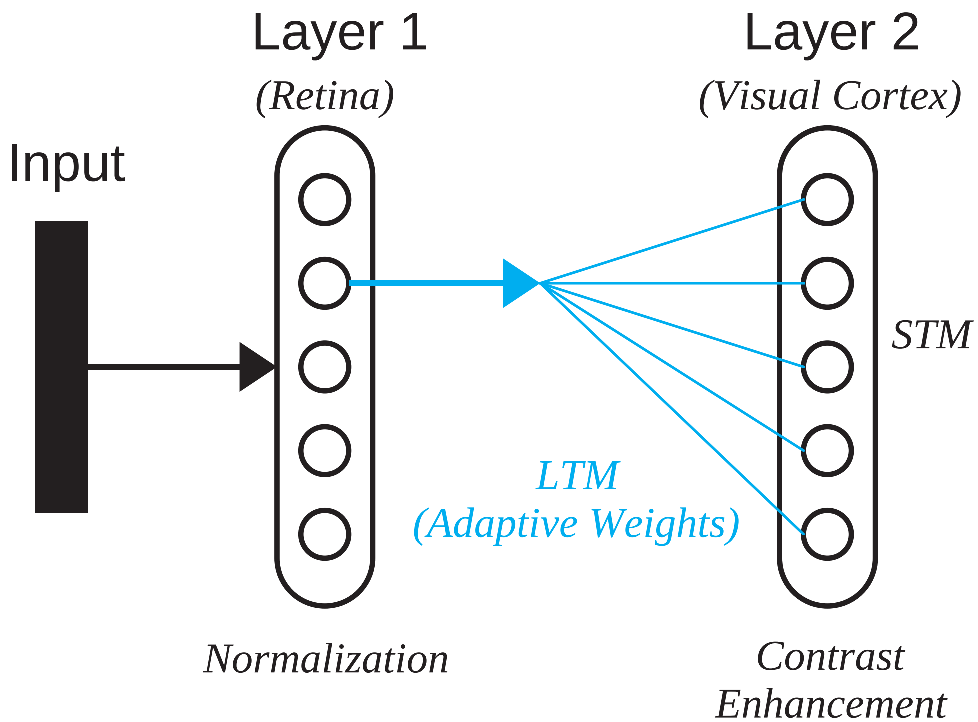
$b^{+} = 1 \qquad b^{-} = 0 \qquad \varepsilon = 1 \qquad p^{-} = 0$

Upper limit will be 1, and lower limit will be 0.





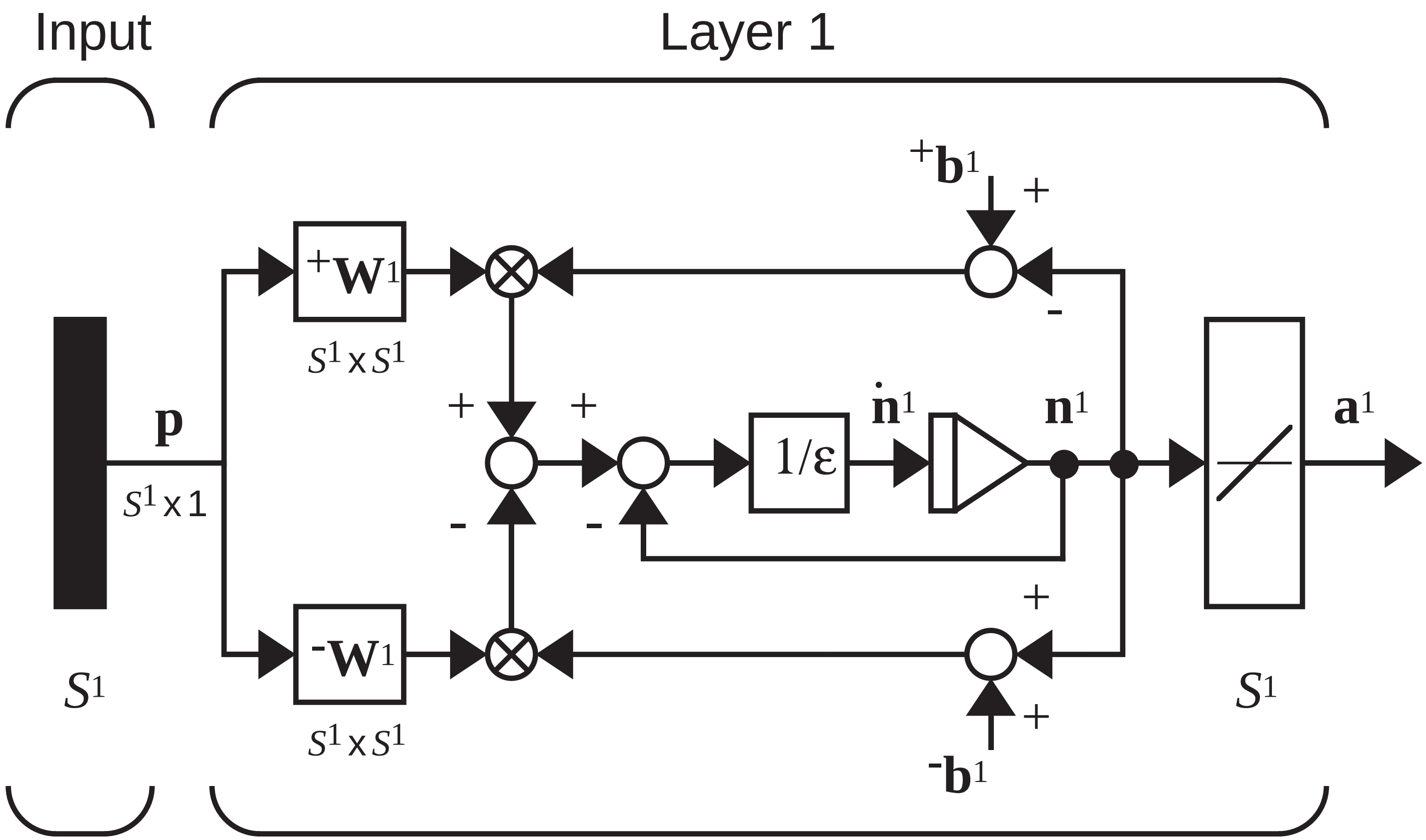
Grossberg Network



LTM - Long Term Memory (Network Weights)

STM - Short Term Memory (Network Outputs)

Layer 1



$$\varepsilon d\mathbf{n}^1/dt = -\mathbf{n}^1 + ({}^+\mathbf{b}^1 - \mathbf{n}^1)[{}^+\mathbf{W}^1]\mathbf{p} - (\mathbf{n}^1 + {}^-\mathbf{b}^1)[{}^-\mathbf{W}^1]\mathbf{p}$$

Operation of Layer 1

$$\varepsilon \frac{d\mathbf{n}^1(t)}{dt} = -\mathbf{n}^1(t) + ({}^{+}\mathbf{b}^1 - \mathbf{n}^1(t))[{}^{+}\mathbf{W}^1]\mathbf{p} - (\mathbf{n}^1(t) + {}^{-}\mathbf{b}^1)[{}^{-}\mathbf{W}^1]\mathbf{p}$$

$${}^{-}\mathbf{b}^1 = \mathbf{0}$$

$${}^{+}b_i^1 = {}^{+}b^1$$

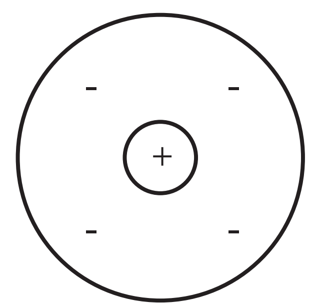
Excitatory Input

$$[{}^{+}\mathbf{W}^1]\mathbf{p} \qquad {}^{+}\mathbf{W}^1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Inhibitory Input

$$[{}^{-}\mathbf{W}^1]\mathbf{p} \qquad {}^{-}\mathbf{W}^1 = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 0 \end{bmatrix}$$

On-Center/Off-Surround Connection Pattern



Normalizes the input while maintaining relative intensities.

Analysis of Normalization

Neuron *i* response:

$$\varepsilon \frac{dn_i^1(t)}{dt} = -n_i^1(t) + ({}^+b^1 - n_i^1(t))p_i - n_i^1(t)\sum_{j \neq i} p_j$$

At steady state:

$$0 = -n_i^1 + ({}^+b^1 - n_i^1)p_i - n_i^1 \sum_{j \neq i} p_j \quad \Rightarrow \quad n_i^1 = \frac{{}^+b^1 p_i}{1 + \sum_{j=1}^{S^1} p_j}$$

Define relative intensity:

$$\bar{p}_i = \frac{p_i}{P} \qquad \text{where} \qquad P = \sum_{j=1}^{S^1} p_j$$

Steady state neuron activity:

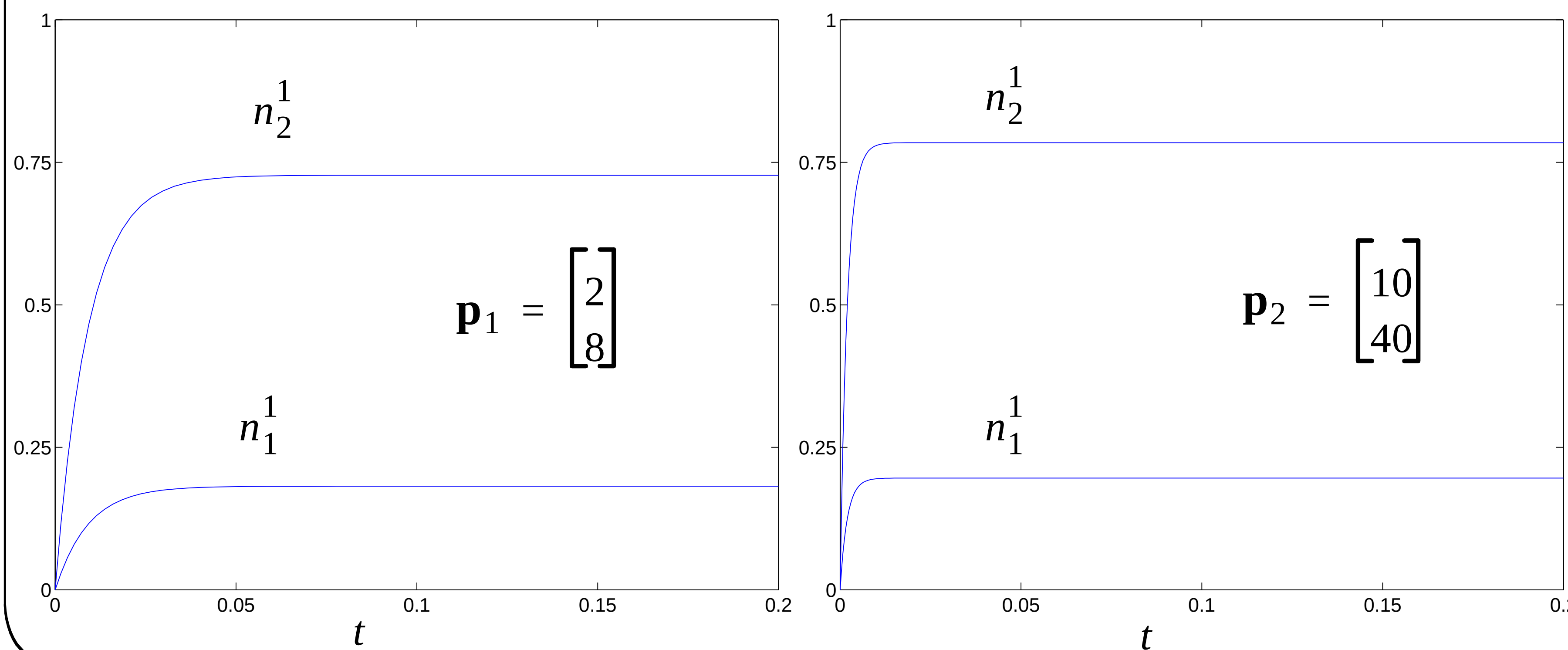
$$n_i^1 = \left(\frac{{}^+b^1 P}{1+P}\right)\bar{p}_i$$

Total activity: $$\sum_{j=1}^{S^1} n_j^1 = \sum_{j=1}^{S^1}\left(\frac{{}^+b^1 P}{1+P}\right)\bar{p}_j = \left(\frac{{}^+b^1 P}{1+P}\right) \le {}^+b^1$$

Layer 1 Example

$$(0.1)\frac{dn_1^1(t)}{dt} = -n_1^1(t) + (1 - n_1^1(t))p_1 - n_1^1(t)p_2$$

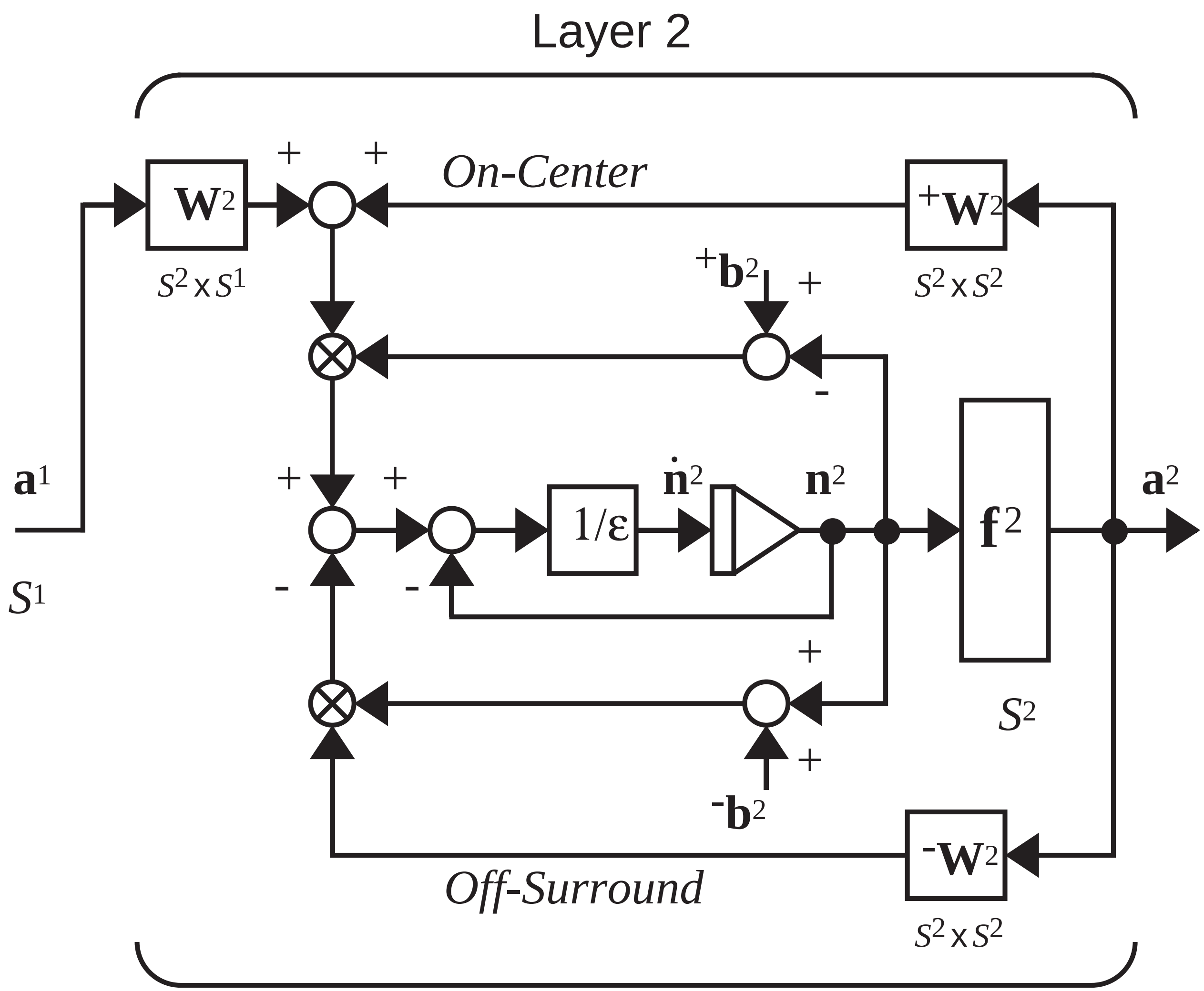
$$(0.1)\frac{dn_2^1(t)}{dt} = -n_2^1(t) + (1 - n_2^1(t))p_2 - n_2^1(t)p_1$$



Characteristics of Layer 1

- The network is sensitive to relative intensities of the input pattern, rather than absolute intensities.
- The output of Layer 1 is a normalized version of the input pattern.
- The on-center/off-surround connection pattern and the nonlinear gain control of the shunting model produce the normalization effect.
- The operation of Layer 1 explains the brightness constancy and brightness contrast characteristics of the human visual system.

Layer 2



$$\varepsilon d\mathbf{n}^2/dt = -\mathbf{n}^2 + ({}^{+}\mathbf{b}^2 - \mathbf{n}^2)\{[{}^{+}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2) + \mathbf{W}^2\mathbf{a}^1\}$$
$$-(\mathbf{n}^2 + {}^{-}\mathbf{b}^2)[{}^{-}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2)$$

Layer 2 Operation

$$\varepsilon \frac{d\mathbf{n}^2(t)}{dt} = -\mathbf{n}^2(t) + ({}^{+}\mathbf{b}^2 - \mathbf{n}^2(t))\{[{}^{+}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2(t)) + \mathbf{W}^2\mathbf{a}^1\} - (\mathbf{n}^2(t) + {}^{-}\mathbf{b}^2)[{}^{-}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2(t))$$

Excitatory Input:

$$\{[{}^{+}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2(t)) + \mathbf{W}^2\mathbf{a}^1\}$$

${}^{+}\mathbf{W}^2 = {}^{+}\mathbf{W}^1$ (On-center connections)

$\mathbf{W}^2$ (Adaptive weights)

Inhibitory Input:

$$[{}^{-}\mathbf{W}^2]\mathbf{f}^2(\mathbf{n}^2(t))$$

${}^{-}\mathbf{W}^2 = {}^{-}\mathbf{W}^1$ (Off-surround connections)

Layer 2 Example

$$\varepsilon = 0.1 \quad {}^{+}\mathbf{b}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad {}^{-}\mathbf{b}^2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad f^2(n) = \frac{10(n)^2}{1+(n)^2} \quad \mathbf{W}^2 = \begin{bmatrix} ({}_1\mathbf{w}^2)^T \\ ({}_2\mathbf{w}^2)^T \end{bmatrix} = \begin{bmatrix} 0.9 & 0.45 \\ 0.45 & 0.9 \end{bmatrix}$$

$$(0.1)\frac{dn_1^2(t)}{dt} = -n_1^2(t) + (1 - n_1^2(t))\left\{ f^2(n_1^2(t)) + \overbrace{({}_1\mathbf{w}^2)^T \mathbf{a}^1}^{\text{Correlation between prototype 1 and input.}} \right\} - n_1^2(t) f^2(n_2^2(t))$$

$$(0.1)\frac{dn_2^2(t)}{dt} = -n_2^2(t) + (1 - n_2^2(t))\left\{ f^2(n_2^2(t)) + \overbrace{({}_2\mathbf{w}^2)^T \mathbf{a}^1}^{\text{Correlation between prototype 2 and input.}} \right\} - n_2^2(t) f^2(n_1^2(t)) .$$

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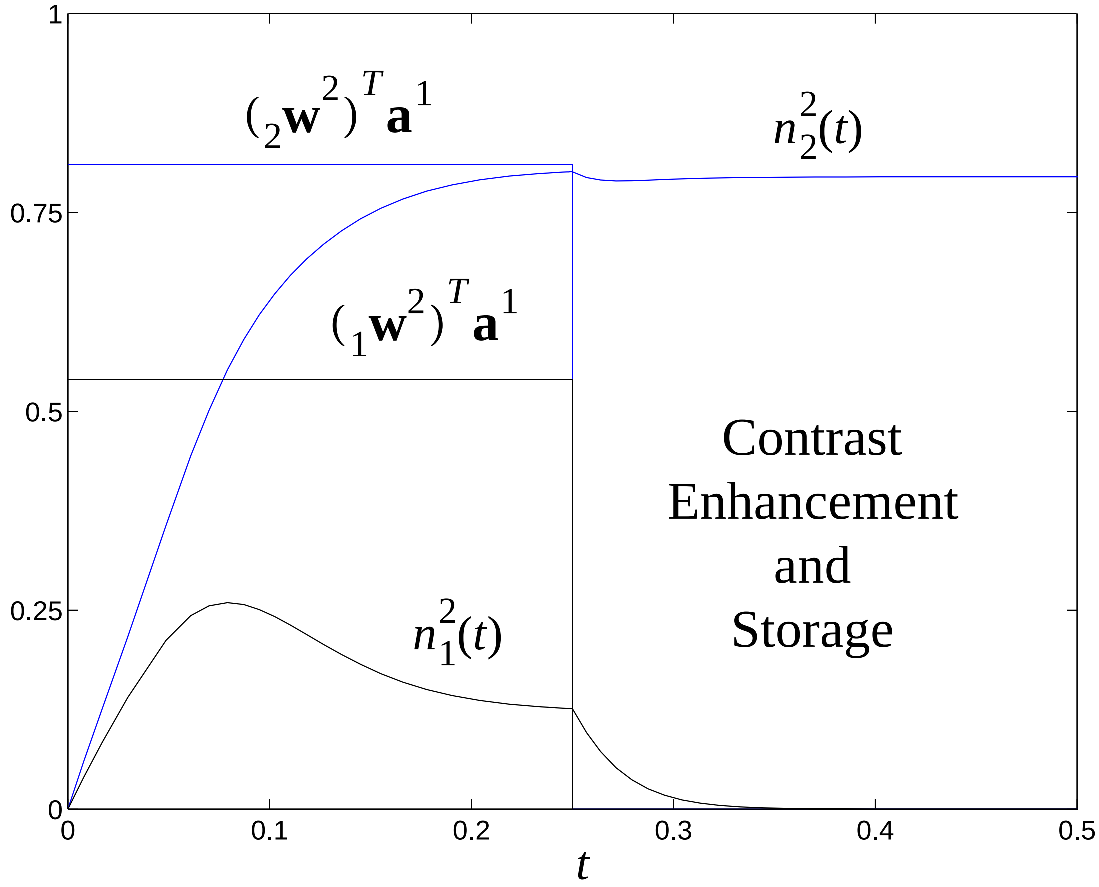
Layer 2 Response

$$\mathbf{a}^1 = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$



Input to neuron 1:

$$({}_1\mathbf{w}^2)^T\mathbf{a}^1 = \begin{bmatrix} 0.9 & 0.45 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = 0.54$$

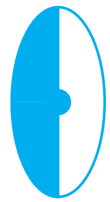
Input to neuron 2:

$$({}_2\mathbf{w}^2)^T\mathbf{a}^1 = \begin{bmatrix} 0.45 & 0.9 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = 0.81$$

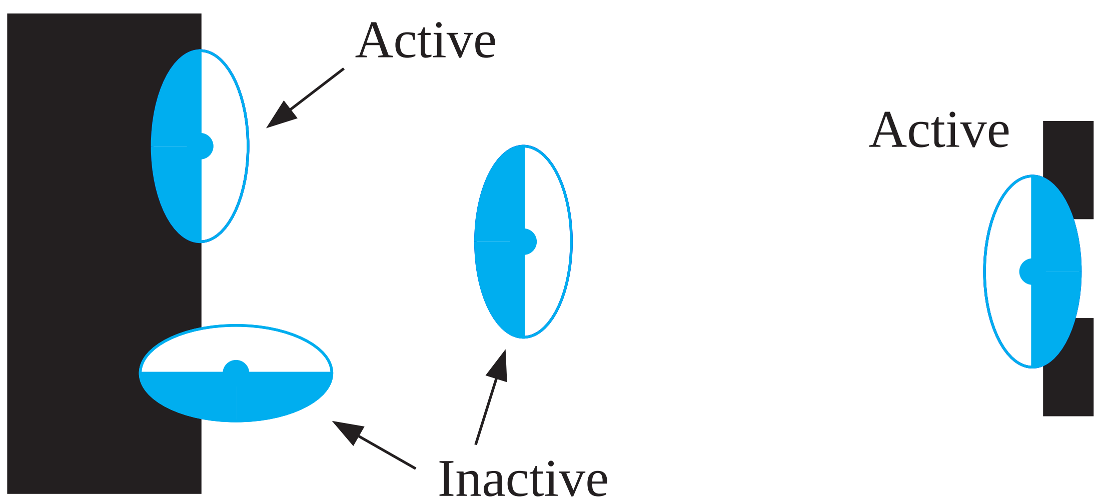
Characteristics of Layer 2

- As in the Hamming and Kohonen networks, the inputs to Layer 2 are the inner products between the prototype patterns (rows of the weight matrix $\mathbf{W}^2$) and the output of Layer 1 (normalized input pattern).
- The nonlinear feedback enables the network to store the output pattern (pattern remains after input is removed).
- The on-center/off-surround connection pattern causes contrast enhancement (large inputs are maintained, while small inputs are attenuated).

Oriented Receptive Field



When an oriented receptive field is used, instead of an on-center/off-surround receptive field, the emergent segmentation problem can be understood.



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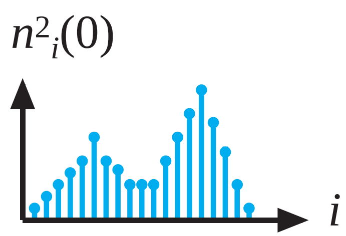
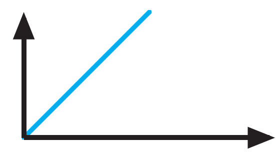
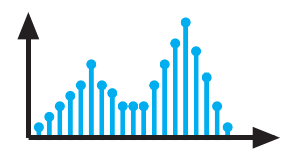
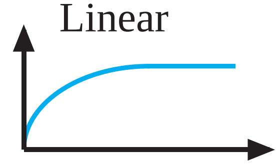
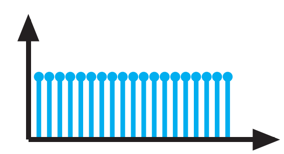
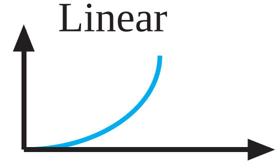
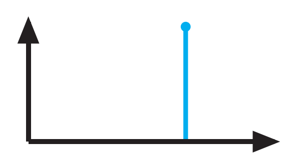
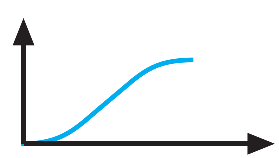
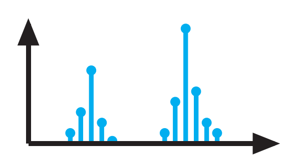
Choice of Transfer Function



$f^2(n)$	Stored Pattern $\mathbf{n}^2(\infty)$	Comments
Linear 		Perfect storage of any pattern, but amplifies noise.
Slower than Linear 		Amplifies noise, reduces contrast.
Faster than Linear 		Winner-take-all, suppresses noise, quantizes total activity.
Sigmoid 		Supresses noise, contrast enhances, not quantized.

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Adaptive Weights

Hebb Rule with Decay

$$\frac{dw^2_{i,j}(t)}{dt} = \alpha\{-w^2_{i,j}(t) + n^2_i(t)n^1_j(t)\}$$

Instar Rule
(Gated Learning)

$$\frac{dw^2_{i,j}(t)}{dt} = \alpha n^2_i(t)\{-w^2_{i,j}(t) + n^1_j(t)\}$$

Learn when $n_i^2(t)$ is active.

Vector Instar Rule

$$\frac{d[{}_i\mathbf{w}^2(t)]}{dt} = \alpha n^2_i(t)\{-[{}_i\mathbf{w}^2(t)] + \mathbf{n}^1(t)\}$$

Example

$$\frac{dw^2_{1,1}(t)}{dt} = n^2_1(t)\{-w^2_{1,1}(t) + n^1_1(t)\}$$

$$\frac{dw^2_{1,2}(t)}{dt} = n^2_1(t)\{-w^2_{1,2}(t) + n^1_2(t)\}$$

$$\frac{dw^2_{2,1}(t)}{dt} = n^2_2(t)\{-w^2_{2,1}(t) + n^1_1(t)\}$$

$$\frac{dw^2_{2,2}(t)}{dt} = n^2_2(t)\{-w^2_{2,2}(t) + n^1_2(t)\}$$

Response of Adaptive Weights

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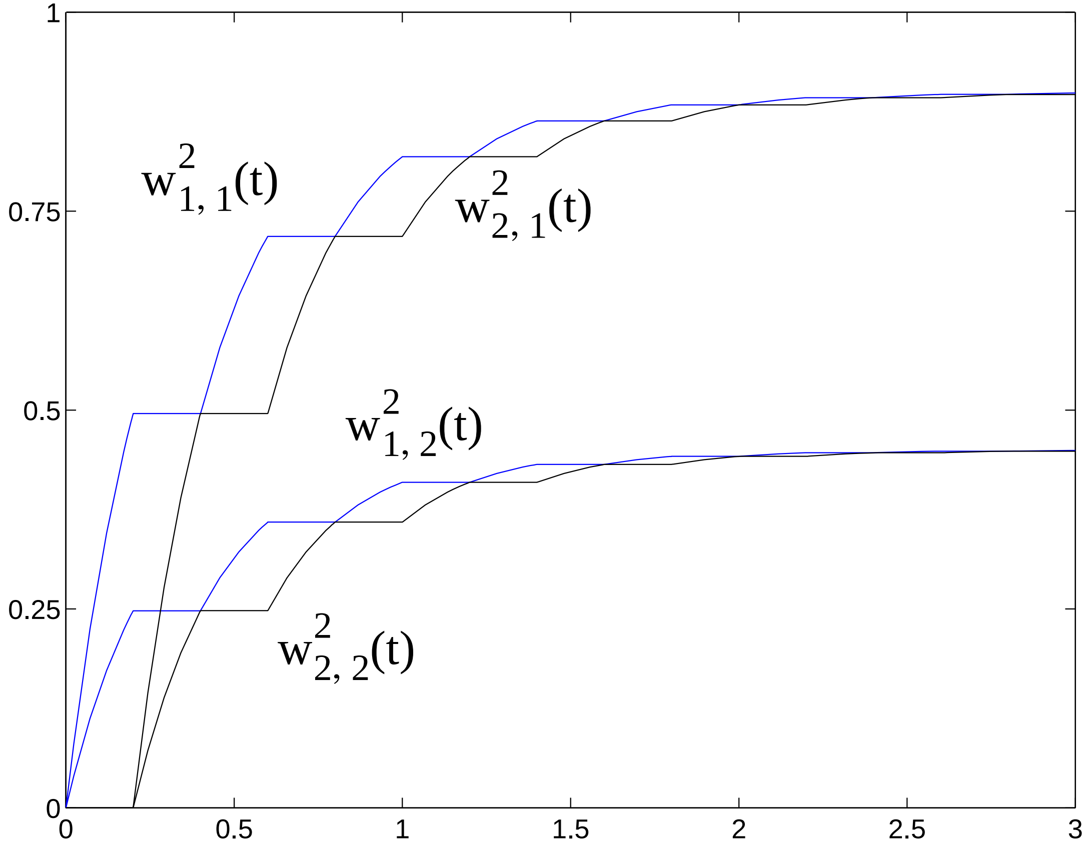
Two different input patterns are alternately presented to the network for periods of 0.2 seconds at a time.

For Pattern 1:

$$\mathbf{n}^1 = \begin{bmatrix} 0.9 \\ 0.45 \end{bmatrix} \qquad \mathbf{n}^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

For Pattern 2:

$$\mathbf{n}^1 = \begin{bmatrix} 0.45 \\ 0.9 \end{bmatrix} \qquad \mathbf{n}^2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



The first row of the weight matrix is updated when $n_1^2(t)$ is active, and the second row of the weight matrix is updated when $n_2^2(t)$ is active.

Relation to Kohonen Law

Grossberg Learning (Continuous-Time)

$$\frac{d[{}_i\mathbf{w}^2(t)]}{dt} = \alpha n_i^2(t)\{-[{}_i\mathbf{w}^2(t)] + \mathbf{n}^1(t)\}$$

Euler Approximation for the Derivative

$$\frac{d[{}_i\mathbf{w}^2(t)]}{dt} \approx \frac{{}_i\mathbf{w}^2(t+\Delta t) - {}_i\mathbf{w}^2(t)}{\Delta t}$$

Discrete-Time Approximation to Grossberg Learning

$${}_i\mathbf{w}^2(t+\Delta t) = {}_i\mathbf{w}^2(t) + \alpha(\Delta t)n_i^2(t)\{-{}_i\mathbf{w}^2(t) + \mathbf{n}^1(t)\}$$

Relation to Kohonen Law

Rearrange Terms

$$_i\mathbf{w}^2(t+\Delta t) = \{1-\alpha(\Delta t)n_i^2(t)\}\,_i\mathbf{w}^2(t) + \alpha(\Delta t)n_i^2(t)\{\mathbf{n}^1(t)\}$$

Assume Winner-Take-All Competition

$$_{i*}\mathbf{w}^2(t+\Delta t) = \{1-\alpha'\}\,_{i*}\mathbf{w}^2(t) + \{\alpha\}'\mathbf{n}^1(t) \quad \text{where} \quad \alpha' = \alpha(\Delta t)n_{i*}^2(t)$$

Compare to Kohonen Rule

$$_{i*}\mathbf{w}(q) = (1-\alpha)\,_{i*}\mathbf{w}(q-1) + \alpha\mathbf{p}(q)$$