

cs155 - z sweedyk

# 3D graphics scene graphs

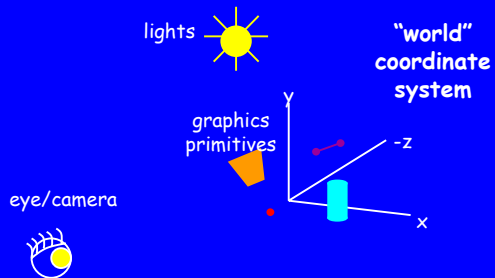
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## overview

- brief overview of 3D
- modeling transforms & homogenous coordinates
- hierarchical coordinates

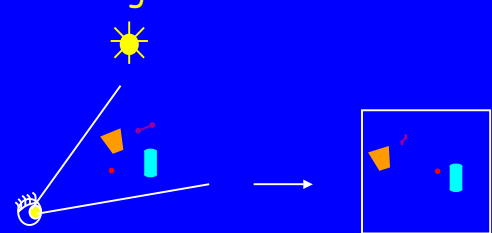
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## 3d scene



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## rendering



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## overview

- brief overview of 3D DONE
- modeling transforms & homogenous coordinates
- hierarchical coordinates

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## today: composing the scene



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## geometric primitives

- point
- line
- polygon
- curved surfaces

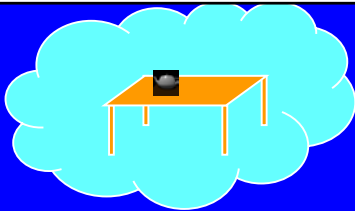
save this for later in  
the semester

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## triangle mesh



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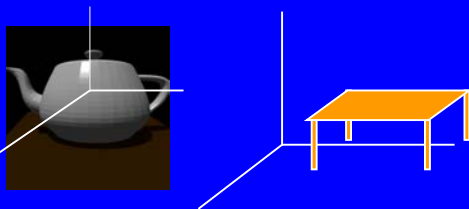
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## 3d model store

receipt
one triangle mesh teapot
one triangle mesh table

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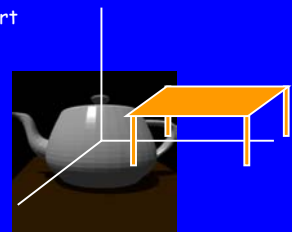
## putting it together



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## putting it together

call Martha Stewart



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## making it fit

transform teapot  
scale by .1  
translate by (20,10,2)

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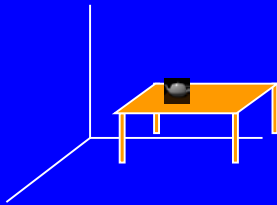
## making it fit

for each triangle in the teapot  
scale by .1  
translate by (20,10,2)

will this work?

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## scale and translate teapot triangles ...



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now rotate it a little

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## transforms

- scale
- rotate
- translate

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## triangle

scale



rotate



translate



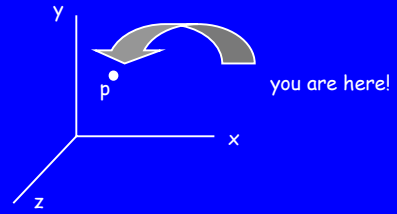
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# triangle



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# points



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# point

scale



call prof gu

rotate



translate



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# linear algebra

- scalars
- vectors

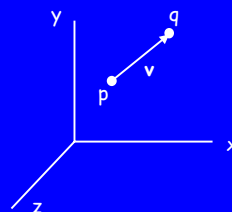
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# scalars: real numbers

3.8  
2.7  
4.1  
-1000.2  
5

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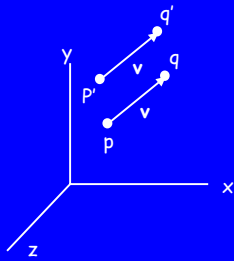
# vector: distance & direction in (3d) space



v: the way you get from p to q

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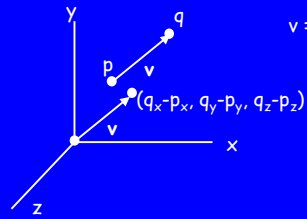
## vector: magnitude & direction in (3d) space



A vector does not have a position in space!

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## naming vectors



$$v = \langle q_x - p_x, q_y - p_y, q_z - p_z \rangle$$

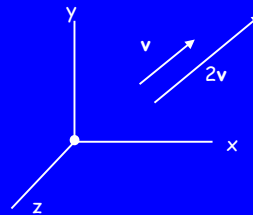
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## linear spaces

- scalars
- vectors
- scalar multiplication
- vector addition

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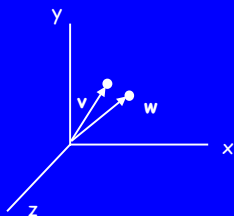
## scalar multiplication



$$2v = \langle 2v_x, 2v_y, 2v_z \rangle$$

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## vector addition

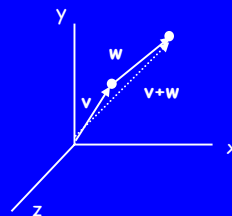


$$v = \langle v_x, v_y, v_z \rangle$$

$$w = \langle w_x, w_y, w_z \rangle$$

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## vector addition

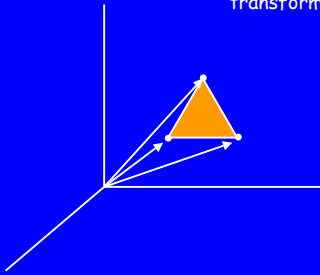


$$v+w = \langle v_x+w_x, v_y+w_y, v_z+w_z \rangle$$

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# triangle

transform vectors!



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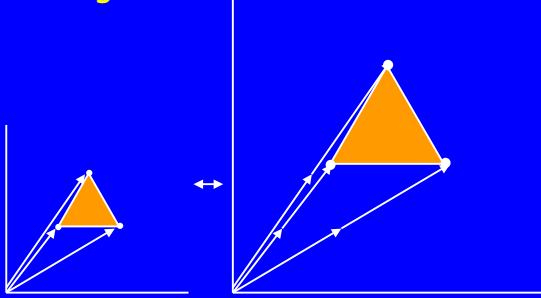
# triangle

scale



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# triangle scale



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# triangle scale



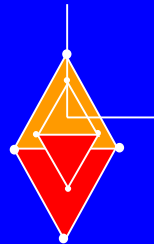
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# triangle scale



9/17/2003

# triangle scale



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## scale

$$\begin{bmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & u \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} sx \\ ty \\ uz \end{bmatrix}$$

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## triangle

scale



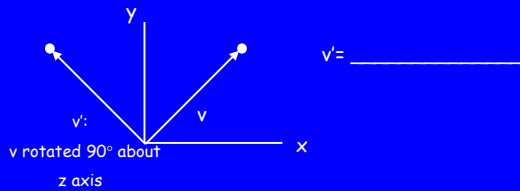
rotate



translate

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## rotating a vector



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## rotation

$$\begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} v_x \cos 90 - v_y \sin 90 \\ v_x \sin 90 + v_y \cos 90 \\ v_z \end{bmatrix}$$

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## rotate about z axis

$$\begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

•the first column specifies what happens to (1,0,0)

•the second column specifies what happens to (0,1,0)

•the third column specifies what happens to (0,0,1)

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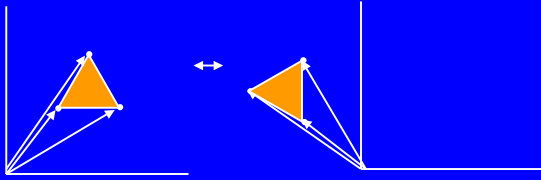
## rotation in 3D about x axis

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

fill in the blanks

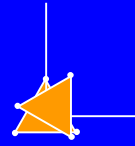
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# triangle rotate



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# triangle rotate



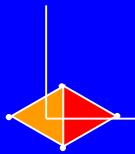
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# triangle rotate



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# triangle rotate



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# triangle

scale



rotate

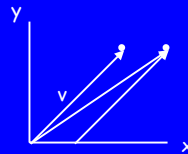


translate



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# translating a point



p translated 2 units to the right

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## operators

- scale } linear
- rotate }
- translate non-linear


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## linear transformation

- $f(\mathbf{v})$  is linear if  $f(\alpha\mathbf{u}+\beta\mathbf{v}) = \alpha f(\mathbf{u})+\beta f(\mathbf{v})$
- translation is not a linear transform:  
define  $T(\mathbf{v}) = \mathbf{v}+\mathbf{w}_0$  where  $\mathbf{w}_0$  is a non-zero vector  
 $T(\mathbf{u}+\mathbf{v}) = \mathbf{u} + \mathbf{v} + \mathbf{w}_0$   
 $T(\mathbf{u})+T(\mathbf{v}) = \mathbf{u}+\mathbf{v}+2\mathbf{w}_0$

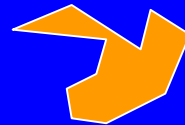
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## composite transform

$$M_1\mathbf{v} \rightarrow M_2(M_1\mathbf{v})$$

$$(M_2M_1)\mathbf{v}$$

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## transform polygon mesh



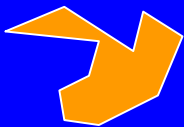
1,000,000  
vertices  
10 transforms



10,000,000  
computations

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## transform polygon mesh



1,000,000  
vertices  
10 transforms  
1 composite transform



1,000,000  
~~10,000,000~~  
computations

9/17/2003

## operators

- scale } linear
- rotate }
- translate non-linear



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## operators

- scale
- rotate
- translate

linear

non-linear

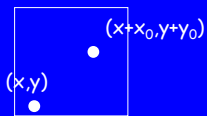


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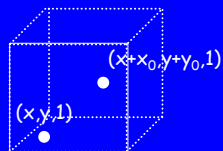
let's step back into 2D for a moment

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## a little trick ... 2D translate



not a 2D linear xfm

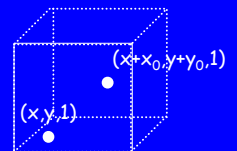


3D linear xfm

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## a little trick ... 2D translate

$$\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+x_0 \\ y+y_0 \\ 1 \end{bmatrix}$$



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## 2D homogenous coordinates

$$(x, y) \longleftrightarrow (x, y, 1)$$

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## scale

$$\begin{bmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sx \\ ty \\ 1 \end{bmatrix}$$

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## rotate

$$\begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \cos \phi - y \sin \phi \\ x \sin \phi + y \cos \phi \\ 1 \end{bmatrix}$$

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## translate

$$\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+x_0 \\ y+y_0 \\ 1 \end{bmatrix}$$

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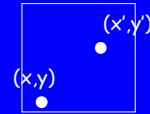
## transform form

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

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## we are not alone...

the parallel universe view of homogenous coordinates

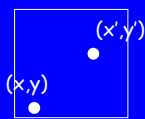


we live in this universe

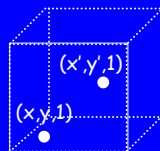
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## we are not alone...

the parallel universe view of homogenous coordinates



we live in this universe

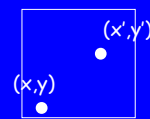


it's not the only one, but it is the only one we can experience!

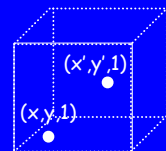
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## and its better not to think about it ...

the parallel universe view of homogenous coordinates



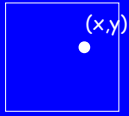
our universe has center (0,0)



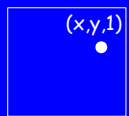
center?

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## 2d and 2d homogenous



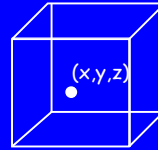
our universe



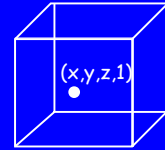
our universe when  
it comes to  
computing modeling  
transforms

9/17/2003

## 3d and 3d homogenous



our universe



our universe when  
it comes to  
computing modeling  
transforms

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## scale

$$\begin{pmatrix} s & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} sx \\ ty \\ uz \\ 1 \end{pmatrix}$$

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## rotate about z axis

$$\begin{pmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \cos \phi - y \sin \phi \\ x \sin \phi + y \cos \phi \\ z \\ 1 \end{pmatrix}$$

rotate about x & y axes are similar

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## translate

$$\begin{pmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x+x_0 \\ y+y_0 \\ z+z_0 \\ 1 \end{pmatrix}$$

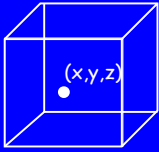
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## transform form

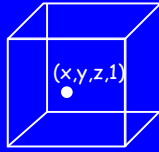
$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z \\ 1 \end{pmatrix}$$

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## 3d and 3d homogenous



our universe



our universe when  
it comes to  
computing modeling  
transforms

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## overview

- brief overview of 3D
- modeling transforms & homogenous coordinates **DONE**
- hierarchical coordinates

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werd! i'm going to make  
a fortune with my  
teapot/table game

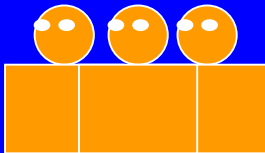


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cool action figure

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## cool action figure



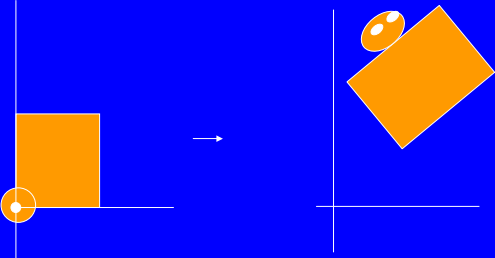
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## cool action figure



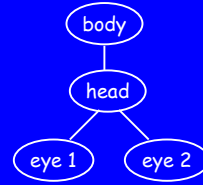
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# transform composition



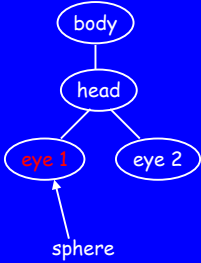
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# scene graph

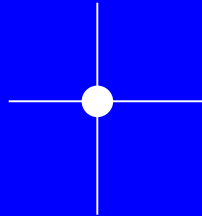


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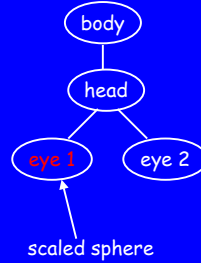
# hierarchical coordinates



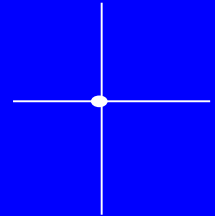
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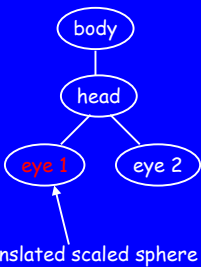
# hierarchical coordinates



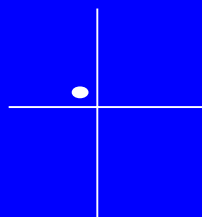
9/17/2003



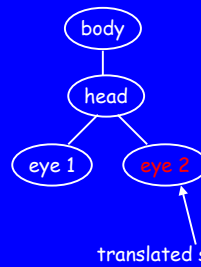
# hierarchical coordinates



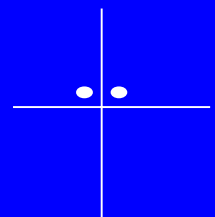
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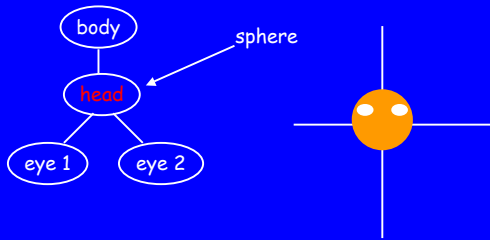
# hierarchical coordinates



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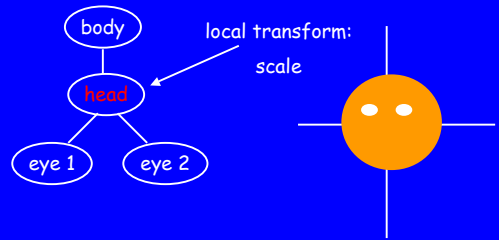


## hierarchical coordinates



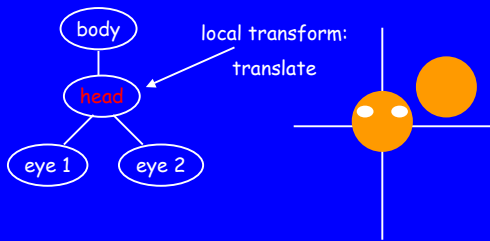
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## hierarchical coordinates



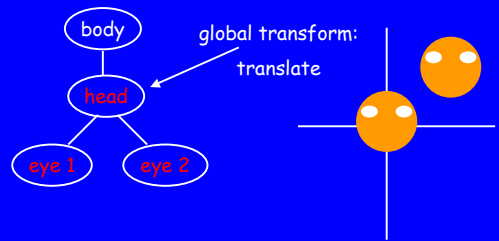
9/17/2003

## hierarchical coordinates



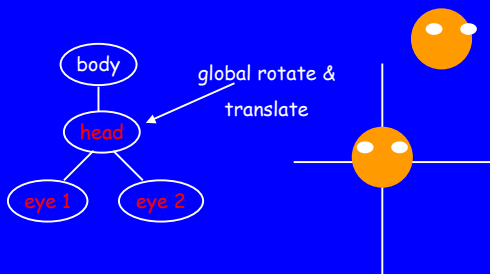
9/17/2003

## hierarchical coordinates



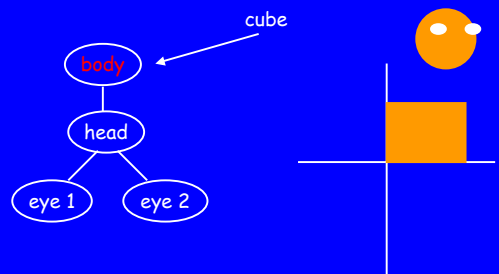
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## hierarchical coordinates



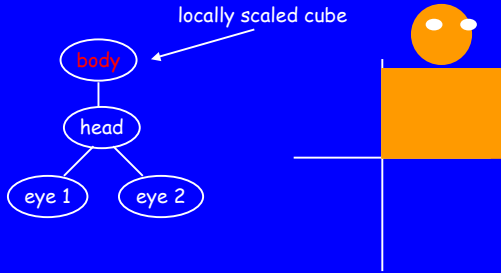
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## hierarchical coordinates



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# hierarchical coordinates

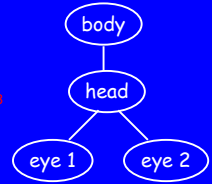


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# the transforms

```

body xfm      M1
{local body scale M1M2
body description
}
head translate wrt body M3
head rotate   M4M5
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```



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# hierarchical coordinates

```

body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```

↑  
more intuitive to specify  
transforms  
in reverse order

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# hierarchical coordinates

```

body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```

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# hierarchical coordinates

```

body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```

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# hierarchical coordinates

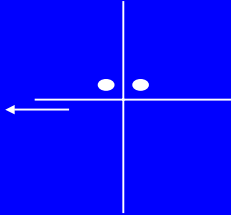
```

body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```

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## hierarchical coordinates

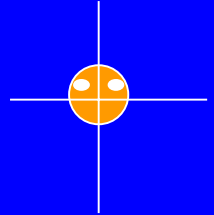
```
body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```



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## hierarchical coordinates

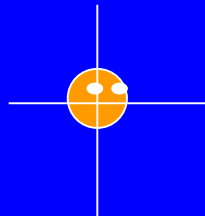
```
body xfm
{local body scale
body description}
head translate wrt body
head rotate
head description ←
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```



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## hierarchical coordinates

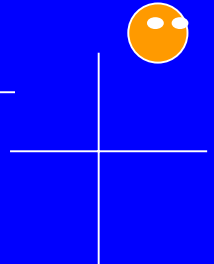
```
body xfm
{local body scale
body description}
head translate wrt body
head rotate ←
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```



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## hierarchical coordinates

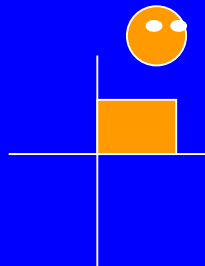
```
body xfm
{local body scale
body description}
head translate wrt body ←
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```



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## hierarchical coordinates

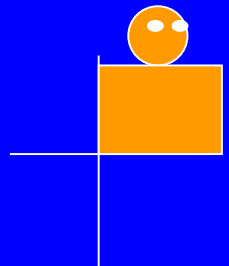
```
body xfm
{local body scale
body description} ←
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```



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## hierarchical coordinates

```
body xfm
{local body scale ←
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
```

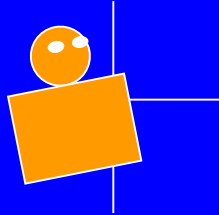


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## hierarchical coordinates

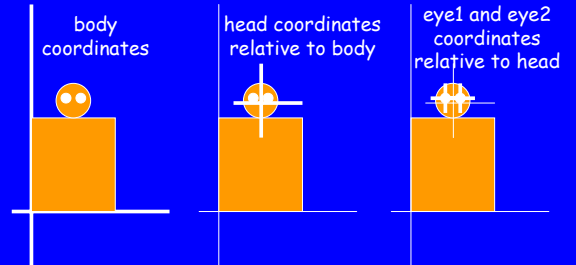
```

body xfm ←
{local body scale
body description}
head translate wrt body
head rotate
head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```



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## hierarchical coordinates

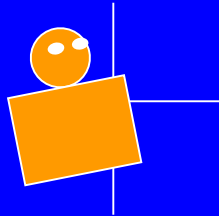


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## now nod

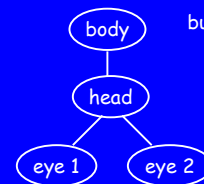
```

body xfm
{local body scale
body description}
head translate wrt body
head rotate
rotate head description
{eye1 translate wrt head
eye1 scale
eye1 description}
{eye2 translate wrt head
eye2 scale
eye2 description}
    
```



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## scene graph



build a scene graph  
for a snowman

9/17/2003

## snowman scene graph

- sphere: radius 1 drawn with center at origin
- cylinder: radius 1 and height 1 sitting on the origin
- cone: bottom radius 1 and height 1 sitting at the origin
- scale (s,t,u)
- rotate (theta, vector)
- translate (dx,dy,dz)

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