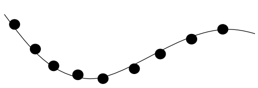
CS155: Modeling

Curves
Z Sweedyk

Overview

- Curves
  - interpolating curves
  - hermitian splines
  - bezier
  - b-splines
- Surfaces
  - splines
  - nurbs
  - surface subdivision

Drawing Curves



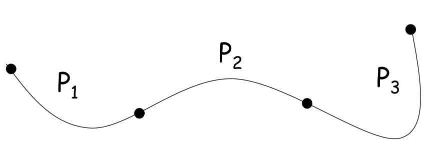
- Sample curve
- Draw line segments between sample points

Representing Curves

How should we represent a curve?

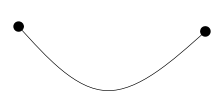
- Flexibility: Can we use the method for a wide range of curves?
- Efficiency: Can we sample it efficiently?
- Usability: Can a user specify it easily?

Complicated Curves



Simple curves connected end-to-end

Simple Curves



How should we represent a simple curve?

- Flexibility
- Efficiency
- Usability
- Boundary constraints: Can we specify continuity (including derivatives) at boundaries?

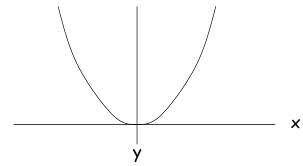
Curve Representation

- Explicit
- Implicit
- Parametric

Explicit

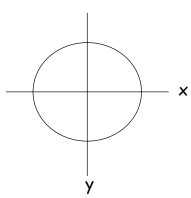
Curve is the trace of a function

Example: $y=x^2/4$



Explicit: flexibility

Many useful curves cannot be represented by explicit functions



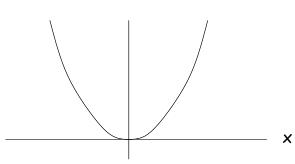
Curve Representation

- ~~Explicit~~
- Implicit
- Parametric

Implicit

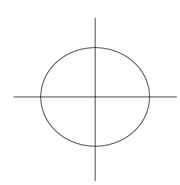
Curve is the zero loci of a function

Example: $f(x,y) = 4y-x^2$



Implicit: more flexibility

$F(x,y)=x^2+y^2-r^2$



But how could we describe a half circle?

Implicit: Efficiency

How can we find the zero loci of an function f(x,y)?

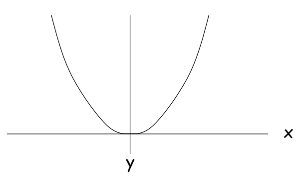
Curve Representation

- ~~Explicit~~
- ~~Implicit~~
- Parametric

Parametric

Curve is the range of a function

Example: $x = 2t,\ y = t^2$



Parametric: tradeoffs

- Flexibility: very expressive, easy to specify portions of curves
- Efficiency: easy to find points on curve
- Boundary conditions: easy to specify
- Usability: not intuitive

Curve Representation

- ~~Explicit~~
- ~~Implicit~~
- ~~Parametric~~

OK -- I give up!

Curve Representation

- ~~Explicit~~
- ~~Implicit~~
- ~~Parametric~~

~~OK -- I give up!~~

Parametric: tradeoffs

- Flexibility: very expressive, easy to specify portions of curves
- Efficiency: easy to find points on curve
- Boundary conditions: easy to specify
- Usability: not intuitive without modeling tools!

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Parametric cubic polynomials

- Polynomials are expressive and can be efficiently computed
- Lower degree polynomials can't express non-planar curves
- Higher degree polynomials
  - Wiggle
  - Computationally more expensive

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Do it yourself demo

Parametric Curves

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Parametric cubic curves

- Interpolating
- Hermitian
- Catmull-Rom
- Bezier
- B-spline

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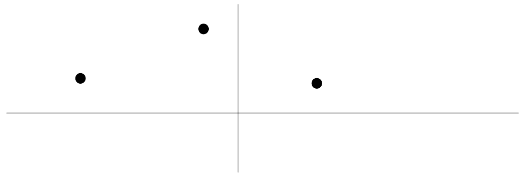
Interpolating polynomials

Give me a lowest degree polynomial curve through these points:



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Interpolation

- Points: (x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)
- Compute: Quadratic polynomials $x(t)$, $y(t)$, $z(t)$ such that

  $(x(i),y(i),z(i)) = (x_i,y_i,z_i)$ for $i=0,1,2$

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Exercise

Give me a parametric quadratic curve through the points (1,0,2), (1,1,1), (2,-1,3)

Let's do x(t) together!

Computing x(t)

We want a quadratic polynomial x(t) such that $x(0)=1$, $x(1)=1$, $x(2)=2$

Step 1

Give me a quadratic polynomial x(t) such that:
- $x(0) = 1$
- $x(1) = 0$
- $x(2) = 0$

Step 2

Give me a quadratic polynomial x(t) such that:
- $x(0) = 0$
- $x(1) = 1$
- $x(2) = 0$

Step 3

Give me a quadratic polynomial x(t) such that:
- $x(0) = 0$
- $x(1) = 0$
- $x(2) = 2$

Step 4

Give me a quadratic polynomial x(t) such that:
- $x(0) = 1$
- $x(1) = 1$
- $x(2) = 2$

Exercise

You do y(t) and z(t)! Write your results on the board.

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General solution

$$x(t) = \Sigma_{i=0\ldots n-1} \frac{x_i\ [\Pi_{j=0\ldots n-1,\ j\neq i}\ (t-j)]}{[\Pi_{j=0\ldots n-1,\ j\neq i}\ (i-j)]}$$

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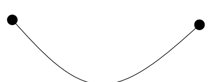
Parametric Curves



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