CS155: Modeling

Curves
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Overview

• Curves
  - interpolating curves
  - hermitian splines
  - bezier
  - b-splines
• Surfaces
  - splines
  - nurbs
  - surface subdivision

Drawing Curves

• Sample curve
• Draw line segments between sample points

Representing Curves

How should we represent a curve?

• Flexibility: Can we use the method for a wide range of curves?
• Efficiency: Can we sample it efficiently?
• Usability: Can a user specify it easily?

Complicated Curves

Simple curves connected end-to-end

Simple Curves

How should we represent a simple curve?

• Flexibility
• Efficiency
• Usability
• Boundary constraints: Can we specify continuity (including derivatives) at boundaries?
Curve Representation

- Explicit
- Implicit
- Parametric

Explicit

Curve is the trace of a function
Example: $y = \frac{x^2}{4}$

Implicit

Curve is the zero loci of a function
Example: $F(x, y) = x^2 + y^2 - r^2$

Explicit: flexibility

Many useful curves cannot be represented by explicit functions

Implicit: more flexibility

But how could we describe a half circle?
Implicit: Efficiency

How can we find the zero loci of a function $f(x,y)$?

Curve Representation

- Explicit
- Implicit
- Parametric

Parametric

Curve is the range of a function

Example: $x = 2t$, $y = t^2$

- Flexibility: very expressive, easy to specify portions of curves
- Efficiency: easy to find points on curve
- Boundary conditions: easy to specify
- Usability: not intuitive

OK -- I give up!
Parametric: tradeoffs

- Flexibility: very expressive, easy to specify portions of curves
- Efficiency: easy to find points on curve
- Boundary conditions: easy to specify
- Usability: not intuitive without modeling tools!

Parametric cubic polynomials

- Polynomials are expressive and can be efficiently computed
- Lower degree polynomials can’t express non-planar curves
- Higher degree polynomials
  - Wiggle
  - Computationally more expensive

Do it yourself demo

Parametric cubic curves

- Interpolating
- Hermitian
- Catmull–Rom
- Bézier
- B-spline

Interpolating polynomials

Give me a lowest degree polynomial curve through these points:

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  .  .  .
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Interpolation

- Points: \((x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)\)
- Compute: Quadratic polynomials \(x(t), y(t), z(t)\) such that
  \[ (x(i), y(i), z(i)) = (x_i, y_i, z_i) \] for \(i = 0, 1, 2\)
Exercise

Give me a parametric quadratic curve through the points
(1,0,2), (1,1,1), (2,-1,3)

Let’s do x(t) together!

Computing x(t)

We want a quadratic polynomial x(t) such that
x(0)=1, x(1)=1, x(2)=2

Step 1

Give me a quadratic polynomial x(t)
such that:
• x(0) = 1
• x(1) = 0
• x(2) = 0

Step 2

Give me a quadratic polynomial x(t)
such that:
• x(0) = 0
• x(1) = 1
• x(2) = 0

Step 3

Give me a quadratic polynomial x(t)
such that:
• x(0) = 0
• x(1) = 0
• x(2) = 2

Step 4

Give me a quadratic polynomial x(t)
such that:
• x(0) = 1
• x(1) = 1
• x(2) = 2
Exercise

You do \( y(t) \) and \( z(t) \)! Write your results on the board.

General solution

\[
\mathbf{x}(t) = \sum_{i=0}^{n-1} \frac{\mathbf{x}_i \prod_{j=0 \neq i}^{n-1} (t-j)}{\prod_{j=0 \neq i}^{n-1} (t-j)}
\]

Parametric Curves

How should we represent a simple curve?

- Flexibility
- Efficiency
- Usability

- Boundary constraints: Can we specify continuity (including derivatives) at boundaries?