Datalog

Logical Rules

Recursion

SQL-99 Recursion

RDB Query Approaches

- Relational Algebra (join, select, proj, …)
- SQL (select … from … where …)
- Relational calculus:
  - Not covered, but capabilities similar to SQL
- Datalog:
  - Relational calculus + defined rules
  - Including recursive rules (not in relational calculus or SQL)

Comparisons

- SQL: select emp, mgr
  from Employee, Manager
  where emp.depno = mgr.depno
- RA: $\prod_{emp,\ mgr}$ (join(Employee, Manager))
- RC: emp, mgr where
  (∃depno) [Employee (emp, depno)
  ∧ Manager(mgr, depno)]
- DL: Answer(emp, mgr) <-
  Employee (emp, depno)
  and Manager(mgr, depno)]

Logic As a Query Language

- If-then logical rules have been used in many systems.
  - Most important today: EII (Enterprise Information Integration).
- Nonrecursive rules are equivalent to the core relational algebra.
- Recursive rules extend relational algebra --- have been used to add recursion to SQL-99.

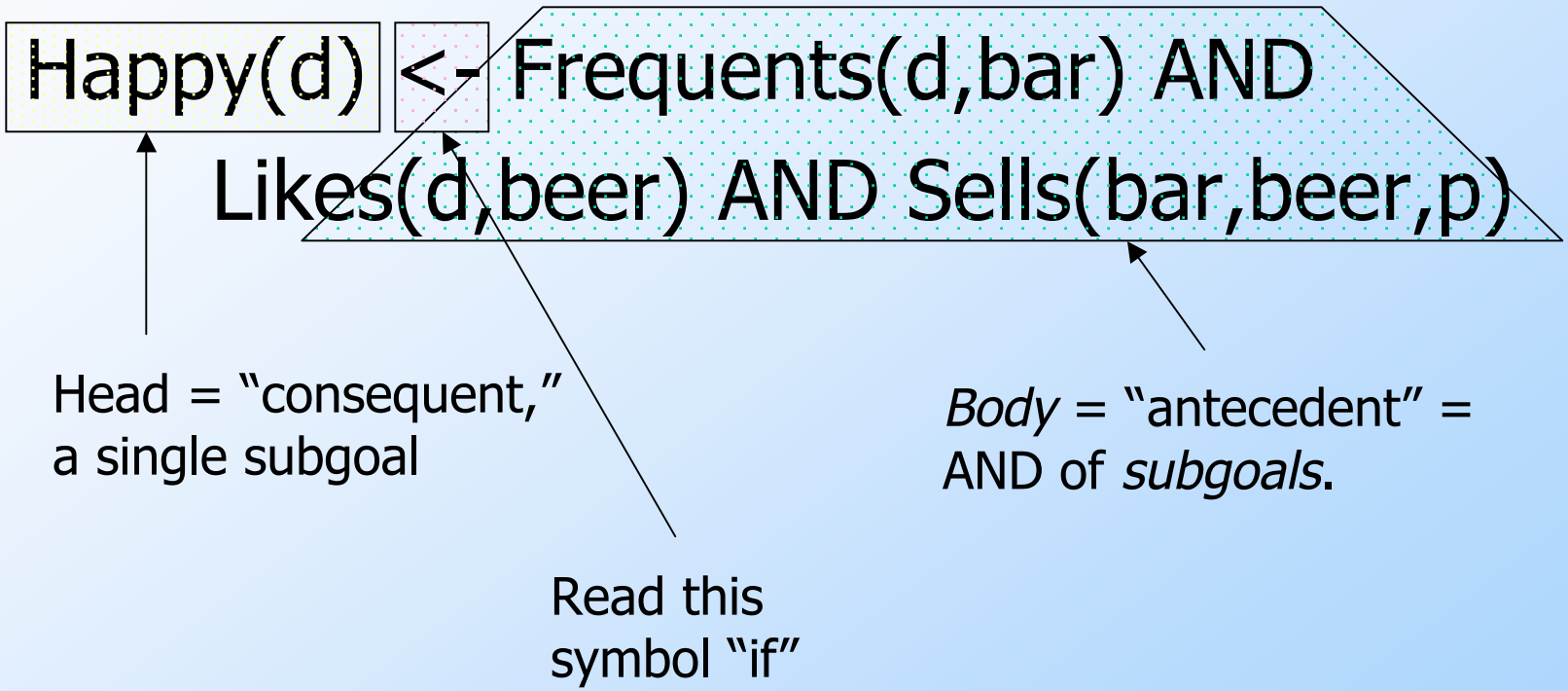
Enterprise Information Integration

- "seamless **integration** of disparate data sources on an **enterprise** scale"

A Logical Rule

- Our first example of a rule uses the relations Frequents(drinker,bar), Likes(drinker,beer), and Sells(bar,beer,price).
- The rule is a query asking for "happy" drinkers --- those that frequent a bar that serves a beer that they like.

Anatomy of a Rule

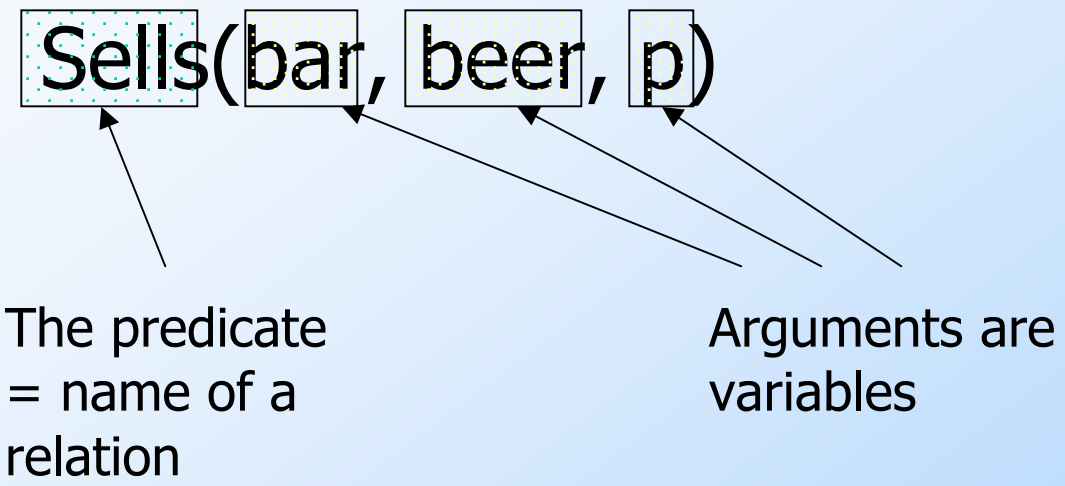


Subgoals Are Atoms

- An *atom* is a *predicate*, or relation name with variables or constants as arguments.
- The head is an atom; the body is the AND of one or more atoms.

- Convention: Predicates begin with a capital, variables begin with lower-case
- In other words, just the OPPOSITE of Prolog's convention.
- Also, some versions of datalog use Prolog's convention.

Example: Atom



Other Datalog/Prolog Differences

- Prolog uses :- where Ullman's version of Datalog uses <-.
- Prolog allows the use of **structures** (general terms) as arguments, whereas as Datalog does not.
- p(f(X, Y, Z), g(Z, Y, X)) :- . . .

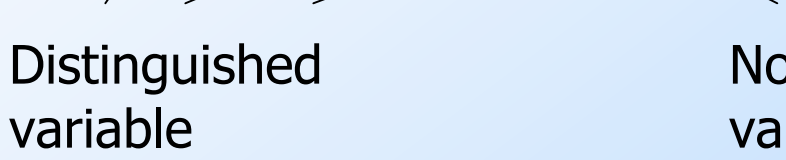
Interpreting Rules

- A variable appearing in the head is called *distinguished* ; otherwise it is *nondistinguished*.

- Rule meaning: The head is true of the distinguished variables if **there exist** values of the nondistinguished variables that make all subgoals of the body true.

Example: Interpretation

Happy(d) <- Frequents(d,bar) AND
Likes(d,beer) AND Sells(bar,beer,p)



Distinguished variable

Nondistinguished variables

Interpretation: drinker d is happy if there exist a bar, a beer, and a price p such that d frequents the bar, likes the beer, and the bar sells the beer at price p.

Arithmetic Subgoals

- In addition to relations as predicates, a predicate for a subgoal of the body can be an arithmetic comparison.
  - We write such subgoals in the usual way, e.g.: $x < y$.

Example: Arithmetic

- A beer is "cheap" if there are at least two bars that sell it for under $2.

```
Cheap(beer) <- Sells(bar1,beer,p1)
               AND Sells(bar2,beer,p2)
               AND p1 < 2.00
               AND p2 < 2.00
               AND bar1 <> bar2
```

Negated Subgoals

- We may put NOT in front of a subgoal, to negate its meaning.
- Example: Think of Arc(a,b) as arcs in a graph.
  - S(x,y) says the graph is not transitive from *x* to *y* ; i.e., there is a path of length 2 from *x* to *y*, but no arc from *x* to *y*.

S(x,y) <- Arc(x,z) AND Arc(z,y)
AND NOT Arc(x,y)

Safe Rules -- Very Important

- A rule is *safe* if:
  1. Each distinguished variable, and
  2. Each variable in an arithmetic subgoal, and
  3. Each variable in a negated subgoal

  also appears in a nonnegated, relational subgoal.
- **We allow only safe rules**.

Example: Unsafe Rules

- Each of the following is unsafe and not allowed:
  1. S(x) <- R(y)
  2. S(x) <- R(y) AND NOT R(x)
  3. S(x) <- R(y) AND x < y
- In each case, an infinity of *x*'s can satisfy the rule, even if *R* is a finite relation.

Algorithms for Applying Rules

- Two approaches:
  1. *Variable-based* : Consider all possible assignments to the variables of the body. If the assignment makes the body true, add that tuple for the head to the result.
  2. *Tuple-based* : Consider all assignments of tuples from the non-negated, relational subgoals. If the body becomes true, add the head's tuple to the result.

Example: Variable-Based --- 1

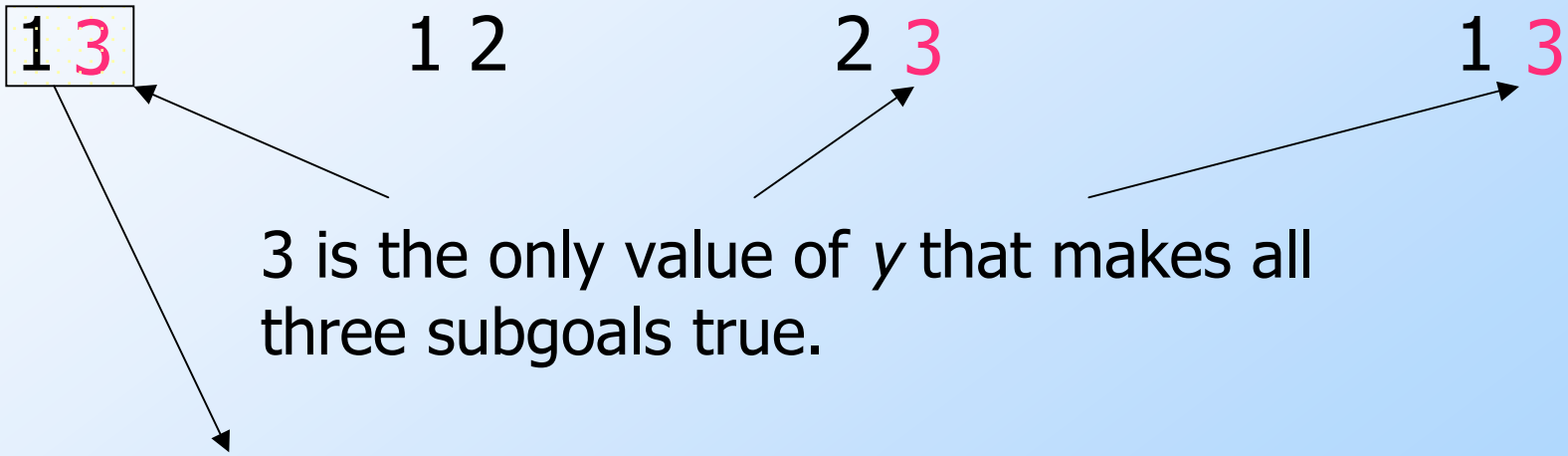
S(x,y) <- Arc(x,z) AND Arc(z,y)
AND NOT Arc(x,y)

- Arc(1,2) and Arc(2,3) are the only tuples in the Arc relation.
- Only assignments to make the first subgoal Arc(x,z) true are:
  1. x = 1; z = 2
  2. x = 2; z = 3

Example: Variable-Based; x=1, z=2

S(x,y) <- Arc(x,z) AND Arc(z,y) AND NOT Arc(x,y)

1 3 1 2 2 3 1 3



3 is the only value of *y* that makes all three subgoals true.

Makes S(1,3) a tuple of the answer

Example: Variable-Based; x=2, z=3

S(x,y) <- Arc(x,z) AND Arc(z,y) AND NOT Arc(x,y)

2 2 3 3 2

Thus, no contribution to the head tuples; S = {(1,3)}

No value of *y* makes Arc(3,y) true.

Tuple-Based Assignment

- Start with the non-negated, relational subgoals only.
- Consider all assignments of tuples to these subgoals.
  - Choose tuples only from the corresponding relations.
- If the assigned tuples give a consistent value to all variables and make the other subgoals true, add the head tuple to the result.

Example: Tuple-Based

S(x,y) <- Arc(x,z) AND Arc(z,y) AND NOT Arc(x,y)

Arc(1,2), Arc(2,3)

- Four possible assignments to first two subgoals:

Arc(x,z)	Arc(z,y)
(1,2)	(1,2)
(1,2)	(2,3)
(2,3)	(1,2)
(2,3)	(2,3)

Only assignment with consistent z-value. Since it also makes NOT Arc(x,y) true, add S(1,3) to result.

Datalog Programs

- A *Datalog program* is a collection of rules.
- In a program, predicates can be either
  1. EDB = *Extensional Database*
     = stored tables.
  2. IDB = *Intensional Database*
     = relations defined by rules.
- Never both! No EDB in heads.

Evaluating Datalog Programs

- As long as there is no recursion, we can pick an order to evaluate the IDB predicates, so that all the predicates in the body of its rules have already been evaluated.
- If an IDB predicate has more than one rule, each rule contributes tuples to its relation.

Example: Datalog Program

- Using EDB Sells(bar, beer, price) and Beers(name, manf), find the manufacturers of beers Joe doesn't sell.

JoeSells(b) <- Sells('Joe"s Bar', b, p)

Answer(m) <- Beers(b,m)
AND NOT JoeSells(b)

Expressive Power of Datalog

- Without recursion, Datalog can express all and only the queries of core relational algebra.
  - The same as SQL select-from-where, **without aggregation and grouping**.
- But with recursion, Datalog can express more than these languages.
- Yet still not Turing-complete.

Exercise: Show how to express, using datalog rules:

- projection
- selection
- natural join
- intersection
- product
- natural join
- difference

Recursive Example

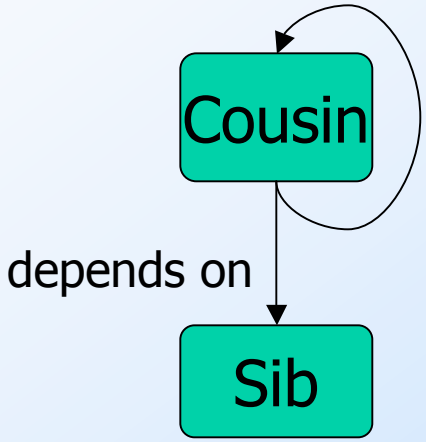
- EDB: Par(c,p) = *p* is a parent of *c*.
- *Generalized* cousins: people with common ancestors one or more generations back:

- Sib(x,y) <- Par(x,p) AND Par(y,p) AND x<>y
- Cousin(x,y) <- Sib(x,y)
- Cousin(x,y) <- Par(x,xp) AND Par(y,yp)
  AND Cousin(xp,yp)

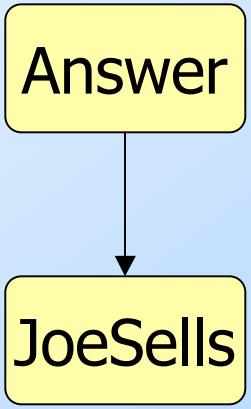
Definition of Recursion

- Form a *dependency graph* whose nodes = IDB predicates.
- Arc *X* -> *Y* if and only if there is a rule with predicate *X* in the head and predicate *Y* in the body.
- Cycle = recursion; no cycle = no recursion.

Example: Dependency Graphs



Recursive

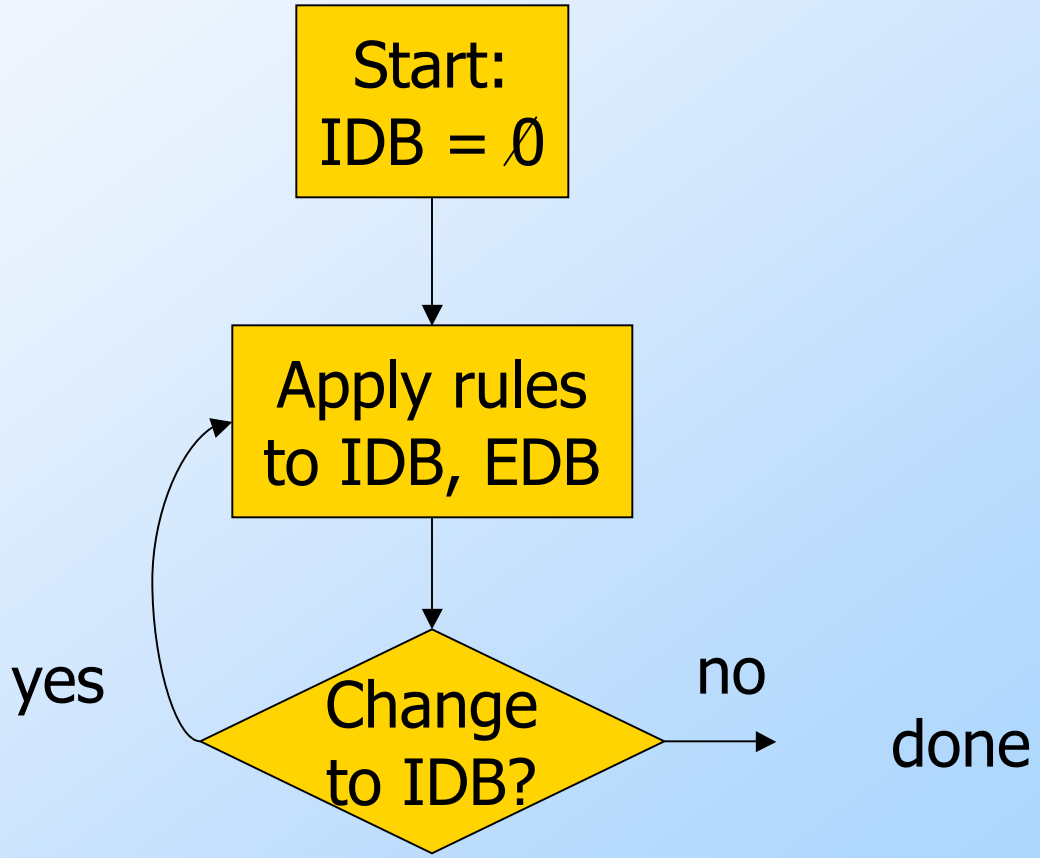


Nonrecursive

Evaluating Recursive Rules

- The following works when there is **no negation**:
  1. Start by assuming all IDB relations are empty.
  2. Repeatedly evaluate the rules using the EDB and the previous IDB, to get a new IDB.
  3. End when no change to IDB.

The "Naïve" Evaluation Algorithm



Example: Evaluation of Cousin

- We'll proceed in rounds to infer Sib facts (red) and Cousin facts (green).
- Remember the rules:

Sib(x,y) <- Par(x,p) AND Par(y,p) AND x<>y

Cousin(x,y) <- Sib(x,y)

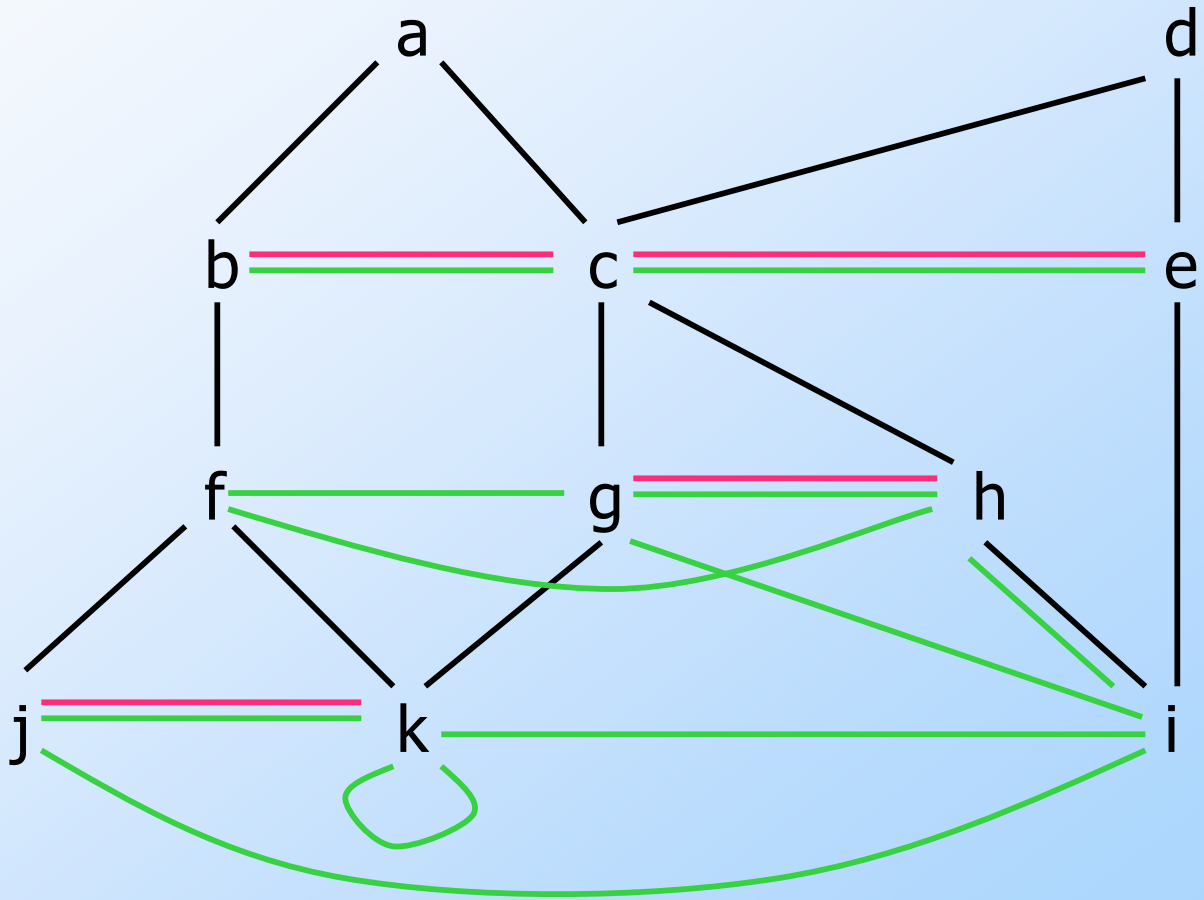
Cousin(x,y) <- Par(x,xp) AND Par(y,yp)
AND Cousin(xp,yp)

Semi-naïve Evaluation

- Since the EDB never changes, on each round we only get new IDB tuples if we use at least one IDB tuple that was obtained on the **previous** round.
- Saves work; lets us avoid rediscovering *most* known facts.
  - A fact could still be derived in a second way.

Par Data: Parent Above Child

Round 1
Round 2
Round 3
Round 4



Recursion Plus Negation

- "Naïve" evaluation doesn't work when there are negated subgoals.
- In fact, negation wrapped in a recursion makes no sense in general.
- Even when recursion and negation are separate, we can have ambiguity about the correct IDB relations.

Stratified Negation

- Stratification is a constraint usually placed on Datalog with recursion and negation.
- **It rules out negation wrapped inside recursion.**
- Gives the sensible IDB relations when negation and recursion are separate.

Problematic Recursive Negation

P(x) <- Q(x) AND NOT P(x)

EDB: Q(1), Q(2)

Initial: P = { }

Round 1: P = {(1), (2)}

Round 2: P = { }

Round 3: P = {(1), (2)}, etc., etc. …

Strata

- Intuitively, the *stratum* of an IDB predicate *P* is the maximum number of negations that can be applied to an IDB predicate used in evaluating *P*.
- Stratified negation = "finite strata."
- Notice in P(x) <- Q(x) AND NOT P(x), we can negate *P* an infinite number of times deriving P(x).

Stratum Graph

- To formalize strata use the *stratum graph* :
  - Nodes = IDB predicates.
  - Arc *A* ->*B* if predicate *A* depends on *B*.
  - **Label this arc "–" if the *B* subgoal is negated.**

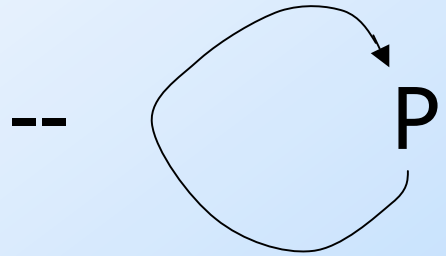
Stratified Negation Definition

- The *stratum* of a node (predicate) is the maximum number of – arcs **on a path** leading from that node (could be ∞).

- A Datalog program is *stratified* if *all* its IDB predicates have finite strata.

Example

P(x) <- Q(x) AND NOT P(x)



Another Example

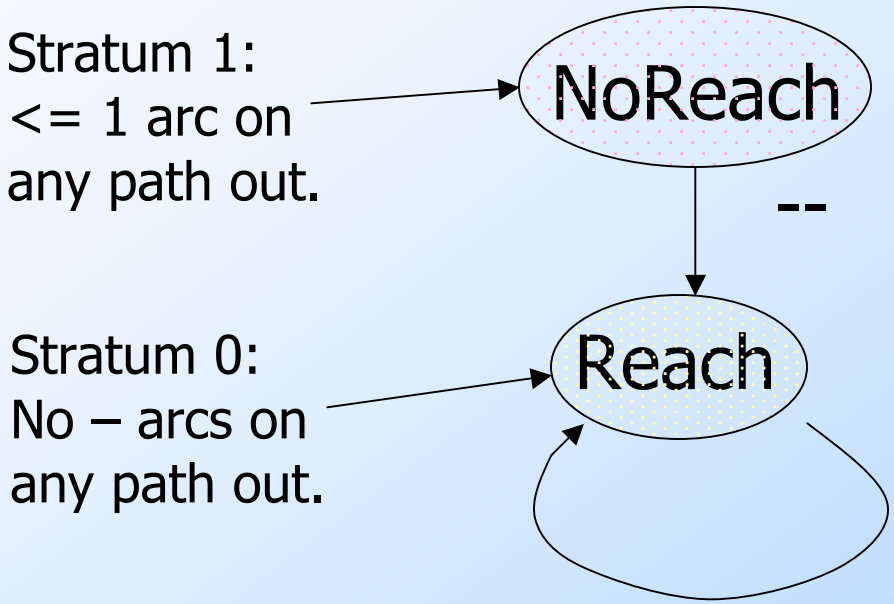
- EDB = Source(x), Target(x), Arc(x,y).
- Rules for "targets not reached from any source":

Reach(x) <- Source(x)

Reach(x) <- Reach(y) AND Arc(y,x)

NoReach(x) <- Target(x)
AND NOT Reach(x)

The Stratum Graph



Models

- A *model* is a choice of IDB relations that, with the given EDB relations makes all rules true regardless of what values are substituted for the variables.
  - Remember: a rule is true whenever its body is false.
  - But if the body is true, then the head must be true as well.

Minimal Models

- When there is no negation, a Datalog program has a **unique minimal model** (one that does not contain any other model).
- But with negation, there can be several minimal models.
- The stratified model is the one that "makes sense."

Example of Models

- P(x) <- Q(x).
- Assume Q = {(a), (b)} is intentional.
- Then P={(a), (b)} is a model.
- So is P = {(a), (b), (c)}.
- P = {(a)} is not a model.
- Evidently, P = {(a), (b)} is the minimal model.

Models with Recursive Rules

- P(1).
- P(x) <- Q(x, y) AND P(y).
- Assume Q(x, y) is intentional, and is true whenever x = y+2, for all y < 97.
- These are models:
  - P(x) for all x in {1, 2, . . ., 99}.
  - P(x) for all x in {1, 3, 5, . . ., 99}.
- The first model is minimal; we can't leave out any element.

Uniqueness of Minimal Model

- **Knaster-Tarski fixed point theorem:**
  If f: L → L is an order-preserving (i.e. monotone) function on a complete lattice, then the set of fixed points of f is a complete lattice.
- In particular, f has a **unique minimal least fixed point**.
- In our case, the lattice is the lattice of tuples of relations (ordered by inclusion).
- The function f is determined from the rules: It determines which additional tuples must be in the relations based upon the right-hand sides of the rules (assuming no negations).
- In logic programming, also called the "closed-world assumption".

The Stratified Model

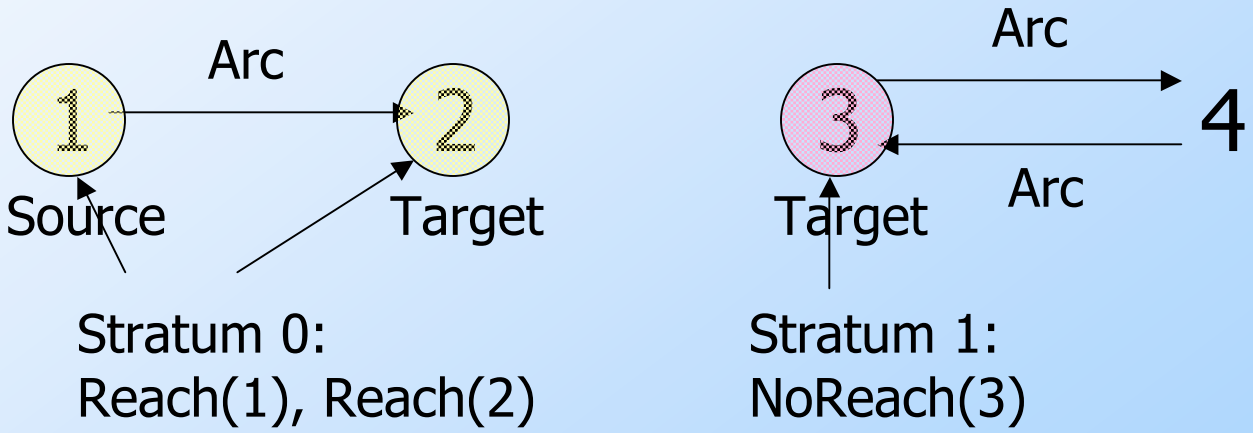
- When the Datalog program is stratified, we can evaluate IDB predicates lowest-stratum-first.
- Once evaluated, treat it as EDB for higher strata.

Example: Multiple Minimal Models in the Non-Monotone Case --- 1

Reach(x) <- Source(x)

Reach(x) <- Reach(y) AND Arc(y,x)

NoReach(x) <- Target(x) AND **NOT** Reach(x)



Example: Multiple Models --- 2

Reach(x) <- Source(x)

Reach(x) <- Reach(y) AND Arc(y,x)

NoReach(x) <- Target(x) AND NOT Reach(x)

Another model! Reach(1), Reach(2), Reach(3), Reach(4); NoReach is empty.

SQL-99 Recursion

- Datalog recursion has inspired the addition of recursion to the SQL-99 standard.
- Trickier, because SQL allows grouping-and-aggregation, which behaves like negation and requires a more complex notion of stratification.

Form of SQL Recursive Queries

WITH

<stuff that looks like Datalog rules>

<an SQL query about EDB, IDB>

Rule =

[RECURSIVE] <name>(<arguments>)

AS <query>

Simple Transitive Closure Example

- Datalog:
  - CLOSURE(x, y) <- R(x, y).
  - CLOSURE(x, y) <- CLOSURE(x, z) AND R(z, y).
- SQL99:
  - WITH RECURSIVE CLOSURE(x, y) AS
    R
    UNION
    SELECT CLOSURE.x, R.y
    FROM CLOSURE, R
    WHERE CLOSURE.y = R.x
    SELECT * FROM CLOSURE;

Example: SQL Recursion --- 1

- Find Sally's cousins, using SQL like the recursive Datalog example.
- Par(child,parent) is the EDB.

```
WITH Sib(x,y) AS
    SELECT p1.child, p2.child
    FROM Par p1, Par p2
    WHERE p1.parent = p2.parent AND
          p1.child <> p2.child;
```

Like Sib(x,y) <-
Par(x,p) AND
Par(y,p) AND
x <> y

Example: SQL Recursion --- 2

```
WITH …
RECURSIVE Cousin(x,y) AS
   (SELECT * FROM Sib)
        UNION
   (SELECT p1.child, p2.child
    FROM Par p1, Par p2, Cousin
    WHERE p1.parent = Cousin.x AND
    p2.parent = Cousin.y);
```

Reflects Cousin(x,y) <- Sib(x,y)

Reflects Cousin(x,y) <- Par(x,xp) AND Par(y,yp) AND Cousin(xp,yp)

Example: SQL Recursion --- 3

- With those definitions, we can add the query, which is about the "temporary view" Cousin(x,y):

```
SELECT y
FROM Cousin
WHERE x = 'Sally';
```

Plan to Explain Legal SQL Recursion

1. Define "monotone," a generalization of "stratified."
2. Generalize stratum graph to apply to SQL.
3. Define proper SQL recursions in terms of the stratum graph.

Monotonicity

- If relation *P* is a function of relation *Q* (and perhaps other relations), we say *P* is *monotone* in *Q* if inserting tuples into *Q* cannot cause any tuple to be deleted from *P*.
- Examples:
  - *P* = *Q* UNION *R*.
  - *P* = $\text{SELECT}_{a=10}(Q)$.

Example: Nonmonotonicity

- If Sells(bar,beer,price) is our usual relation, then the result of the query:

```
SELECT AVG(price)
FROM Sells
WHERE bar = 'Joe''s Bar';
```

is **not monotone** in Sells.

- Inserting a Joe's-Bar tuple into Sells usually changes the average price and thus deletes the old average price.

SQL Stratum Graph --- 2

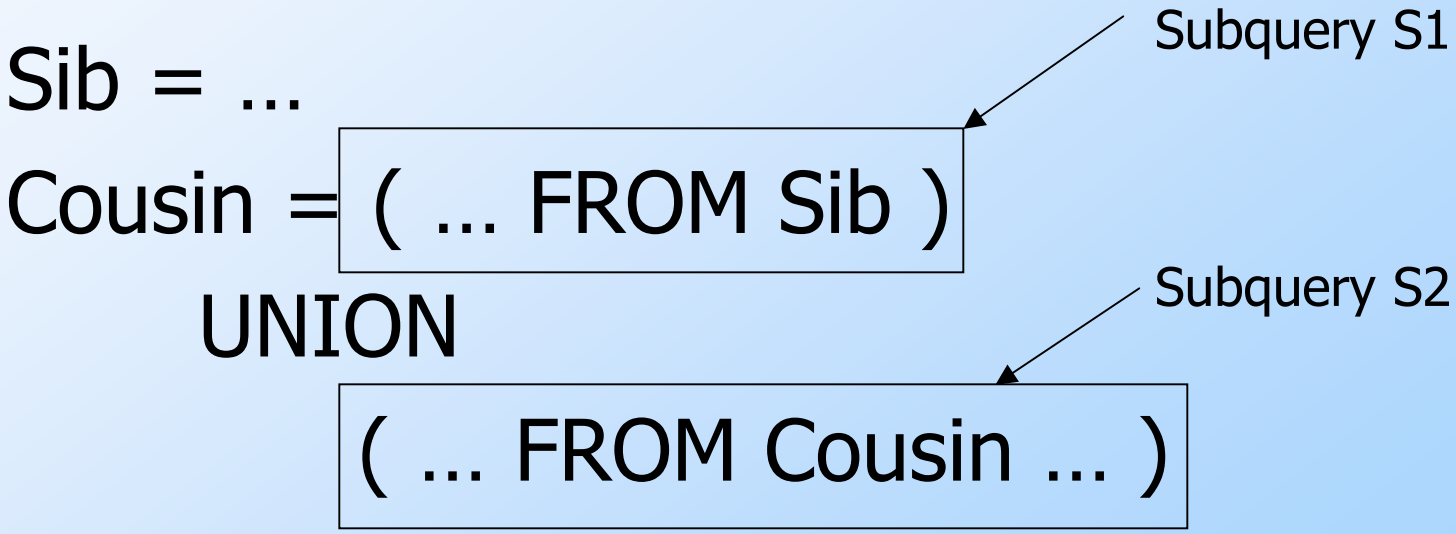
- Nodes =
  1. IDB relations declared in WITH clause.
  2. Subqueries in the body of the "rules."
     - Includes subqueries at any level of nesting.

SQL Stratum Graph --- 2

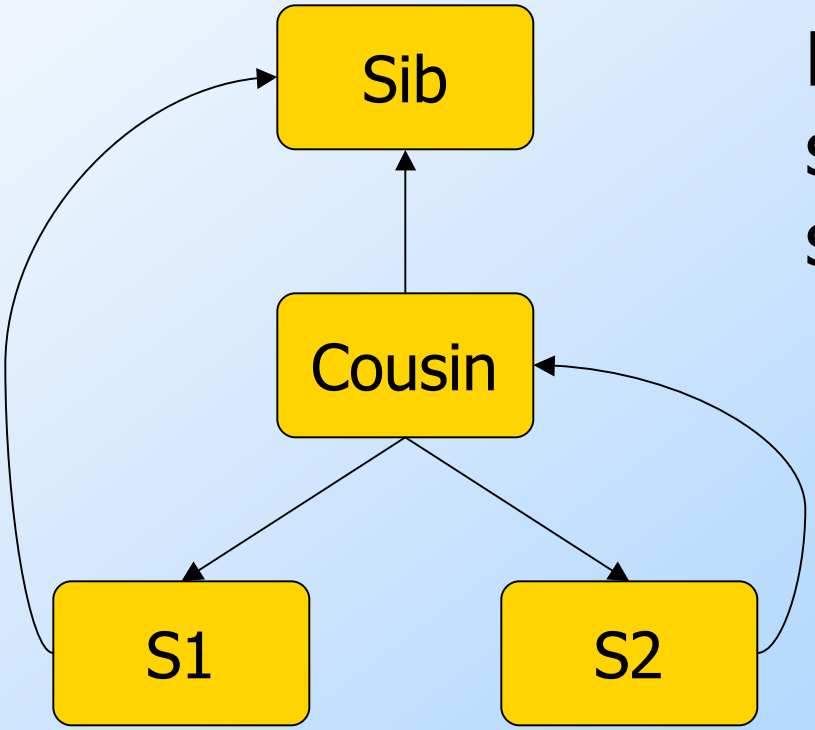
- Arcs *P* ->*Q* :
  1. *P* is a rule head and *Q* is a relation in the FROM list (not of a subquery).
  2. *P* is a rule head and *Q* is an immediate subquery of that rule.
  3. *P* is a subquery, and *Q* is a relation in its FROM or an immediate subquery (like 1 and 2).
- Put "–" on an arc if *P* is not monotone in *Q*.
- Stratified SQL = finite #'s of –'s on paths.

Example: Stratum Graph

- In our Cousin example, the structure of the rules was:



The Graph



No "–" at all, so surely stratified.

Nonmonotone Example

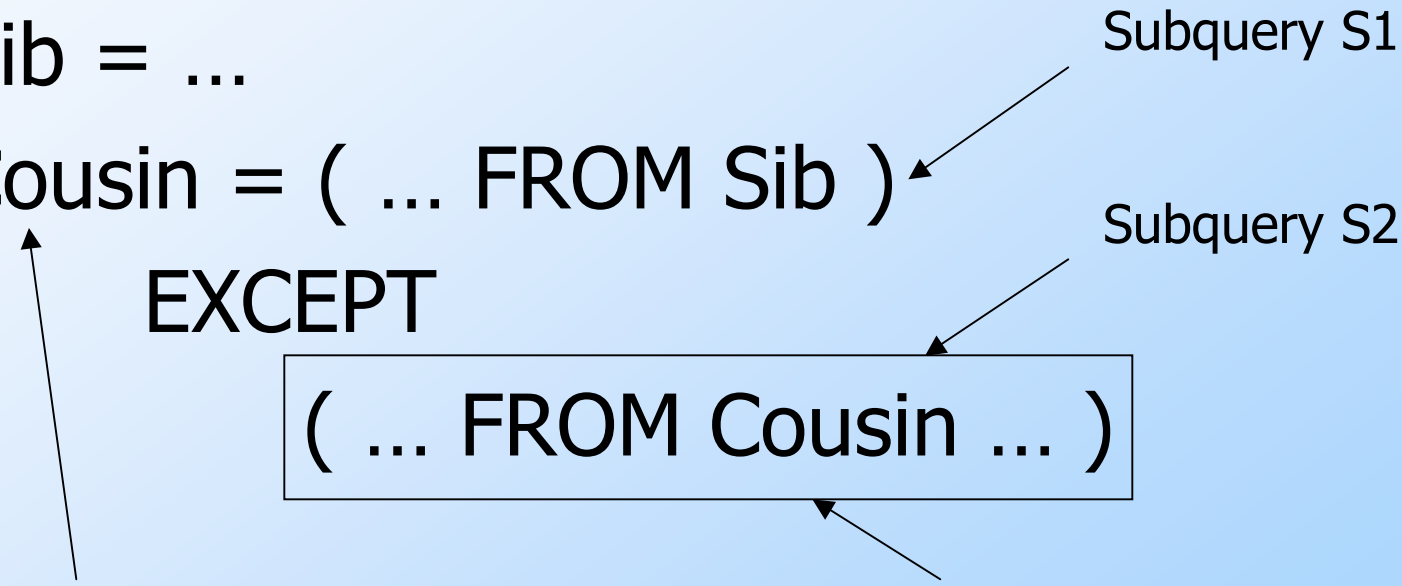
- Change the UNION in the Cousin example to EXCEPT:

Sib = …

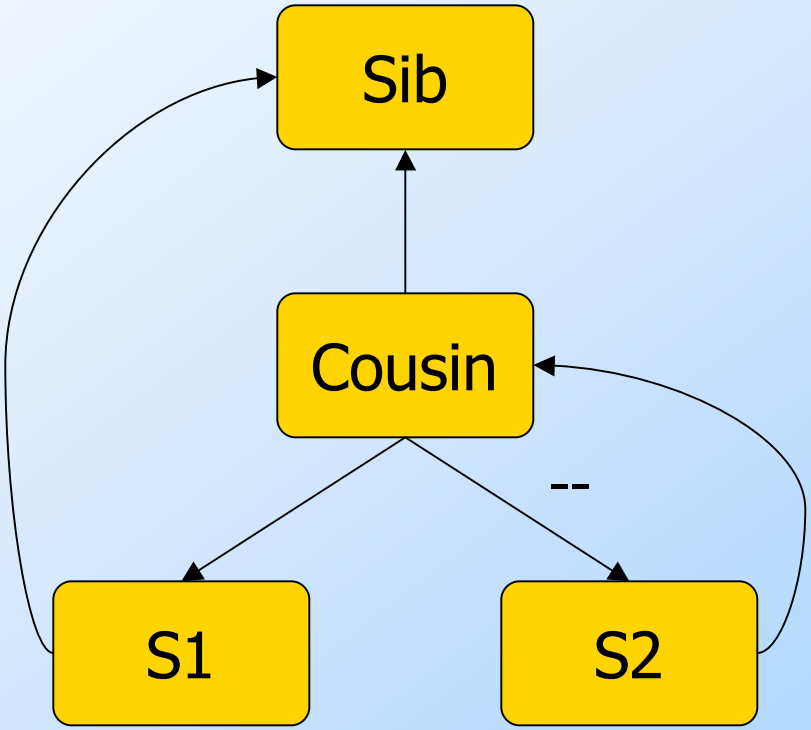
Cousin = (… FROM Sib)

EXCEPT

(… FROM Cousin …)



The Graph



An infinite number of –'s exist on cycles involving Cousin and S2.

NOT Doesn't Mean Nonmonotone

- Not every NOT means the query is nonmonotone.
  - We need to consider each case separately.
- Example: Negating a condition in a WHERE clause just changes the selection condition.
  - But all selections are monotone.

Example: Revised Cousin

```
RECURSIVE Cousin AS
   (SELECT * FROM Sib)
      UNION
   (SELECT p1.child, p2.child
    FROM Par p1, Par p2, Cousin
    WHERE p1.parent = Cousin.x AND
       NOT (p2.parent = Cousin.y)
   );
```

Revised subquery S2

The only difference

S2 Still Monotone in Cousin

- Intuitively, adding a tuple to Cousin cannot delete from S2.
- All former tuples in Cousin can still work with Par tuples to form S2 tuples.
- In addition, the new Cousin tuple might even join with Par tuples to add to S2.