The Relational Data Model

Functional Dependencies

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Functional Dependencies

- *X* -> *A* is an assertion about a relation *R* that whenever two tuples of *R* agree on all the attributes of *X*, then they must also agree on the attribute *A*.
  - Say "*X* -> *A* holds in *R*."
  - Notice convention: …,*X*, *Y*, *Z* represent sets of attributes; *A*, *B*, *C*,… represent single attributes.
  - Convention: no set formers in sets of attributes, just *ABC*, rather than {*A*,*B*,*C* }.

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Example

- Drinkers(name, addr, beersLiked, manf, favBeer).
- Reasonable FD's to assert:
  1. name -> addr
  2. name -> favBeer
  3. beersLiked -> manf

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Example Data

name	addr	beersLiked	manf	favBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	Voyager	WickedAle	Pete's	WickedAle
Spock	Enterprise	Bud	A.B.	Bud

Because name -> addr

Because name -> favBeer

Because beersLiked -> manf

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FD's With Multiple Attributes

- No need for FD's with > 1 attribute on right.
  - But sometimes convenient to combine FD's as a shorthand.
  - Example: name -> addr and name -> favBeer become name -> addr favBeer
- > 1 attribute on left may be essential.
  - Example: bar beer -> price

Keys of Relations

- *K* is a ***key*** for relation *R* if:
  1. Set *K* functionally determines all attributes of *R*
  2. For no proper subset of *K* is (1) true.

- If *K* satisfies (1), but *perhaps* not (2), then *K* is a ***superkey***.

- Note **E/R keys** have no requirement for **minimality**, as in (2) for relational keys.

Example

- Consider relation

  Drinkers(name, addr, beersLiked, manf, favBeer).

- {name, beersLiked} is a superkey because together these attributes determine all the other attributes.
  - name -> addr favBeer
  - beersLiked -> manf

Example, Cont.

- {name, beersLiked} is a **key** because neither {name} nor {beersLiked} is a superkey.
  - name **–/–>** manf;
    (using **–/–>** to mean "does not functionally determine")
  - beersLiked **–/–>** addr.
- In this example, there are no other keys, but lots of superkeys.
  - Any superset of {name, beersLiked}.

E/R and Relational Keys

- Keys in E/R are **properties of entity sets**.
- Keys in relations are properties of **relations.**
- Usually, one tuple corresponds to one entity, so then the ideas are the same.
- But in poor relational designs, one entity set can become several relations, so E/R keys and relational keys are different.

Example Data

name	addr	beersLiked	manf	favBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	Voyager	WickedAle	Pete's	WickedAle
Spock	Enterprise	Bud	A.B.	Bud

Relational key = **name beersLiked**

But in E/R, name is a key for Drinkers, and beersLiked is a key for Beers.

Note: 2 tuples for Janeway entity and 2 tuples for Bud entity.

From Where Do Keys Come?

1. We could simply assert a key *K*. Then the only [?] FD's are *K* –> *A* for all atributes *A*, and *K* turns out to be the only key obtainable from the FD's.

2. We could assert FD's and deduce the keys by systematic exploration.
   - E/R gives us FD's from entity-set keys and many-one relationships.

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FD's From "Physics"

- While most FD's come from E/R keyness and many-one relationships, some are really physical laws.
- Example: "no two courses can meet in the same room at the same time" tells us: `hour room -> course`.
- [In software engineering, these would be examples of **"business rules"**.]

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Inferring FD's: Motivation

- In order to **design** relation schemas well, we often need to tell what FD's hold in a relation.
- We are given FD's $X_1 \rightarrow A_1$, $X_2 \rightarrow A_2$,…, $X_n \rightarrow A_n$, and we want to know whether an FD $Y \rightarrow B$ must hold in any relation that satisfies the given FD's.

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Example: Transitivity Rule

- If $A \rightarrow B$ and $B \rightarrow C$ hold, surely $A \rightarrow C$ holds as well.

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[A Viewpoint of Attributes]

- Whereas one could think of attributes as "labels" on columns, a more mathematical viewpoint is to think of them as **functions**:

  attribute : set-of-tuples -> set of values

- If a is an attribute and t is a tuple, then a(t) is the value of that attribute in tuple t.

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[FD Under the Function Interpretation]

- $A_1\ A_2 \dots A_n \rightarrow B$
  means: For any tuples t, t' in a relation corresponding to the relation schema:

  If for all i, $A_i(t) = A_i(t')$

  then $B(t) = B(t')$.

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Using the Functional Characterization

- Show that A –> B and B –> C imply A –> C.
- A –> B means
  □ t, t’ A(t) = A(t’) implies B(t) = B(t’).
- B –> C means
  □ t, t’ B(t) = B(t’) implies C(t) = C(t’).
- Putting these together, we get:
  □ t, t’ A(t) = A(t’) implies C(t) = C(t’)
  so A –> C as well.

Inference Test

- To test if *Y* –> *B*, start assuming two tuples agree in all attributes of *Y*.
- Use the given FD’s to infer that these tuples must also agree in certain other attributes.
- If B is eventually found to be one of these attributes, then *Y* –> *B* is true; otherwise, the two tuples, with any forced equalities form a two-tuple relation that proves *Y* –> *B* does not follow from the given FD’s.

Closure Test

- An easier way to test is to compute the *closure* of Y, denoted Y^+:
  - Basis: $Y^+ = Y$.
  - Induction: Look for an FD's left side X that is a subset of the current Y^+. If the FD is $X \rightarrow A$, add A to Y^+.

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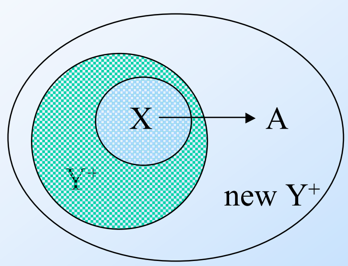

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Finding All Implied FD's

- Motivation: "normalization," the process where we break a relation schema into two or more schemas [by **projecting** it onto the attributes of the new schemas].
- Example: *ABCD* with FD's *AB* ->*C*, *C* ->*D*, and *D* ->*A*.
  - Decompose into *ABC*, *AD*. What FD's hold in *ABC* ?
  - Not only *AB* ->*C*, but also *C* ->*A* !

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Basic Idea

- To know what FD's hold **in a projection**, we start with given FD's and find all FD's that follow from given ones.
- Then, **restrict** to those FD's that involve only attributes of the projected schema.

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Simple, Exponential Algorithm

1. For each set of attributes X, compute X^+.
2. Add $X \to A$ for all A in $X^+ - X$.
3. However, drop $XY \to A$ whenever we discover $X \to A$.
   - Because $XY \to A$ follows from $X \to A$.
4. Finally, use only FD's involving **projected** attributes.

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A Few Tricks

- Never need to compute the closure of the empty set or of the set of all attributes.
- If we find X^+ = all attributes, don't bother computing the closure of any supersets of X.

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Example

- *ABC* with FD's $A \to B$ and $B \to C$. Project onto *AC*.
  - $A^+ = ABC$; yields $A \to B$, $A \to C$.
    - We do not need to compute AB^+ or AC^+.
  - $B^+ = BC$; yields $B \to C$.
  - $C^+ = C$; yields nothing.
  - $BC^+ = BC$; yields nothing.

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Example, Continued

- Resulting FD's: $A \to B$, $A \to C$, and $B \to C$.
- Projection onto *AC* : $A \to C$.
  - Only FD that involves a subset of $\{A, C\}$.

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A Geometric View of FD's

- Imagine the set of all instances of a particular relation.
- That is, all finite sets of tuples that have the proper number of components.
- Each instance is a point in this space.

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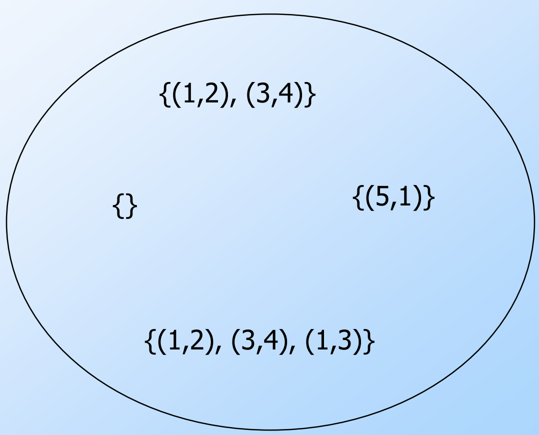
Example: R(A,B)



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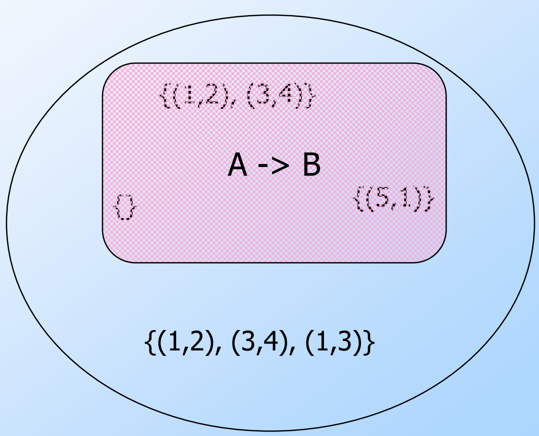
An FD is a Subset of Instances

- For each FD *X* -> *A* there is a subset of all instances that satisfy the FD.
- We can represent an FD by a region in the space.
- *Trivial FD*: an FD that is represented by the entire space.
  - Example: *A* -> *A*.

Example: A -> B for R(A,B)



Representing Sets of FD's

- If each FD is a set of relation instances, then a collection of FD's corresponds to the intersection of those sets.
  - Intersection = all instances that satisfy all of the FD's.

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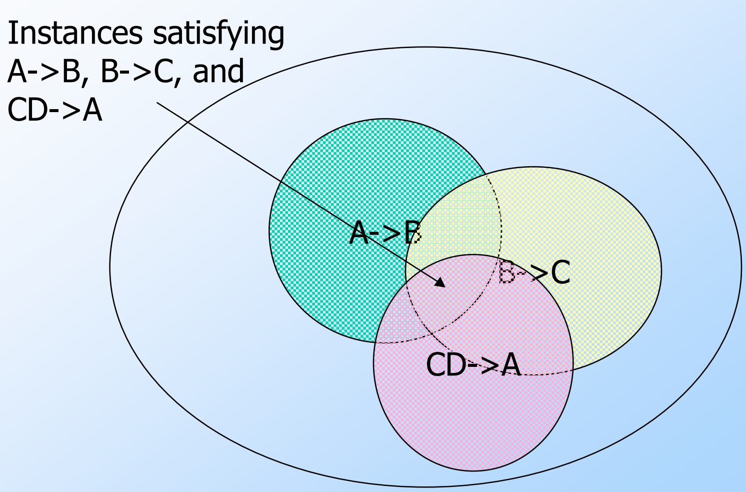
Example



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Implication of FD's

- If an FD Y -> B follows from FD's X_1 -> A_1,…, X_n -> A_n, then the region in the space of instances for Y -> B must include the intersection of the regions for the FD's X_i -> A_i.
  - That is, every instance satisfying all the FD's X_i -> A_i surely satisfies Y -> B.
  - But an instance could satisfy Y -> B, yet not be in this intersection.

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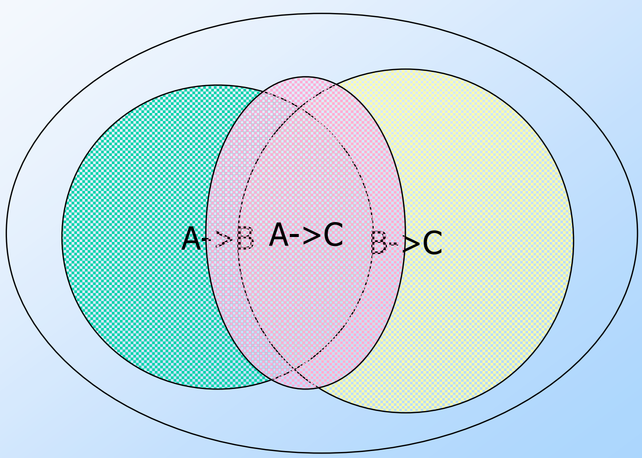
Example



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