The Relational Data Model

Functional Dependencies

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Functional Dependencies

- *X* -> *A* is an assertion about a relation *R* that whenever two tuples of *R* agree on all the attributes of *X*, then they must also agree on the attribute *A*.
  - Say "*X* -> *A* holds in *R*."
  - Notice convention: …,*X*, *Y*, *Z* represent sets of attributes; *A*, *B*, *C*,… represent single attributes.
  - Convention: no set formers in sets of attributes, just *ABC*, rather than {*A*,*B*,*C* }.

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Example

- Drinkers(name, addr, beersLiked, manf, favBeer).
- Reasonable FD's to assert:
  1. name -> addr
  2. name -> favBeer
  3. beersLiked -> manf

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Example Data

name	addr	beersLiked	manf	favBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	Voyager	WickedAle	Pete's	WickedAle
Spock	Enterprise	Bud	A.B.	Bud

Because name -> addr

Because name -> favBeer

Because beersLiked -> manf

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FD's With Multiple Attributes

- No need for FD's with > 1 attribute on right.
  - But sometimes convenient to combine FD's as a shorthand.
  - Example: name -> addr and name -> favBeer become name -> addr favBeer
- > 1 attribute on left may be essential.
  - Example: bar beer -> price

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Keys of Relations

- *K* is a ***key*** for relation *R* if:
  1. Set *K* functionally determines all attributes of *R*
  2. For no proper subset of *K* is (1) true.
- If *K* satisfies (1), but *perhaps* not (2), then *K* is a ***superkey***.
- Note **E/R keys** have no requirement for **minimality**, as in (2) for relational keys.

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Example

- Consider relation

  Drinkers(name, addr, beersLiked, manf, favBeer).

- {name, beersLiked} is a superkey because together these attributes determine all the other attributes.
  - name -> addr favBeer
  - beersLiked -> manf

Example, Cont.

- {name, beersLiked} is a **key** because neither {name} nor {beersLiked} is a superkey.
  - name **–/–>** manf;
    (using **–/–>** to mean "does not functionally determine")
  - beersLiked **–/–>** addr.
- In this example, there are no other keys, but lots of superkeys.
  - Any superset of {name, beersLiked}.

E/R and Relational Keys

- Keys in E/R are **properties of entity sets**.
- Keys in relations are properties of **relations**.
- Usually, one tuple corresponds to one entity, so then the ideas are the same.
- But in poor relational designs, one entity set can become several relations, so E/R keys and relational keys are different.

Example Data

name	addr	beersLiked	manf	favBeer
Janeway	Voyager	Bud	A.B.	WickedAle
Janeway	Voyager	WickedAle	Pete's	WickedAle
Spock	Enterprise	Bud	A.B.	Bud

Relational key = **name beersLiked**
But in E/R, name is a key for Drinkers, and beersLiked is a key for Beers.
Note: 2 tuples for Janeway entity and 2 tuples for Bud entity.

From Where Do Keys Come?

1. We could simply assert a key *K*. Then the only [?] FD's are *K* –> *A* for all atributes *A*, and *K* turns out to be the only key obtainable from the FD's.
2. We could assert FD's and deduce the keys by systematic exploration.
   - E/R gives us FD's from entity-set keys and many-one relationships.

FD's From "Physics"

- While most FD's come from E/R keyness and many-one relationships, some are really physical laws.
- Example: "no two courses can meet in the same room at the same time" tells us: `hour room –> course`.
- [In software engineering, these would be examples of **"business rules"**.]

Inferring FD's: Motivation

- In order to **design** relation schemas well, we often need to tell what FD's hold in a relation.
- We are given FD's X_1 –> A_1, X_2 –> A_2 ,…, X_n –> A_n , and we want to know whether an FD *Y* -> *B* must hold in any relation that satisfies the given FD's.

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Example: Transitivity Rule

- If *A* -> *B* and *B* -> *C* hold, surely *A* -> *C* holds as well.

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[A Viewpoint of Attributes]

- Whereas one could think of attributes as "labels" on columns, a more mathematical viewpoint is to think of them as **functions**:

  attribute : set-of-tuples -> set of values

- If a is an attribute and t is a tuple, then a(t) is the value of that attribute in tuple t.

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[FD Under the Function Interpretation]

- A_1 A_2 … A_n –> B
  means: For any tuples t, t' in a relation corresponding to the relation schema:

  If for all i, A_i (t) = A_i (t')

  then B(t) = B(t') .

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Using the Functional Characterization

- Show that A –> B and B –> C imply A –> C.
- A –> B means
  ∀ t, t' A(t) = A(t') implies B(t) = B(t').
- B –> C means
  ∀ t, t' B(t) = B(t') implies C(t) = C(t').
- Putting these together, we get:
  ∀ t, t' A(t) = A(t') implies C(t) = C(t')
  so A –> C as well.

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Inference Test

- To test if *Y* –> *B*, start assuming two tuples agree in all attributes of *Y*.
- Use the given FD's to infer that these tuples must also agree in certain other attributes.
- If B is eventually found to be one of these attributes, then *Y* –> *B* is true; otherwise, the two tuples, with any forced equalities form a two-tuple relation that proves *Y* –> *B* does not follow from the given FD's.

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Closure Test

- An easier way to test is to compute the *closure* of Y, denoted Y^+:
  - Basis: $Y^+ = Y$.
  - Induction: Look for an FD's left side X that is a subset of the current Y^+. If the FD is $X \to A$, add A to Y^+.

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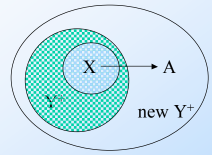

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Finding All Implied FD's

- Motivation: "normalization," the process where we break a relation schema into two or more schemas [by **projecting** it onto the attributes of the new schemas].
- Example: *ABCD* with FD's $AB \to C$, $C \to D$, and $D \to A$.
  - Decompose into *ABC*, *AD*. What FD's hold in *ABC* ?
  - Not only $AB \to C$, but also $C \to A$!

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Basic Idea

- To know what FD's hold **in a projection**, we start with given FD's and find all FD's that follow from given ones.
- Then, **restrict** to those FD's that involve only attributes of the projected schema.

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Simple, Exponential Algorithm

1. For each set of attributes X, compute X^+.
2. Add $X \to A$ for all A in $X^+ - X$.
3. However, drop $XY \to A$ whenever we discover $X \to A$.
   - Because $XY \to A$ follows from $X \to A$.
4. Finally, use only FD's involving **projected** attributes.

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A Few Tricks

- Never need to compute the closure of the empty set or of the set of all attributes.
- If we find X^+ = all attributes, don't bother computing the closure of any supersets of X.

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Example

- *ABC* with FD's *A* ->*B* and *B* ->*C.* Project onto *AC.*
  - A^+=*ABC* ; yields *A* ->*B, A* ->*C.*
    - We do not need to compute AB^+ or AC^+.
  - B^+=*BC* ; yields *B* ->*C.*
  - C^+=*C* ; yields nothing.
  - BC^+=*BC* ; yields nothing.

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Example, Continued

- Resulting FD's: *A* ->*B, A* ->*C,* and *B* –>*C.*
- Projection onto *AC* : *A* ->*C.*
  - Only FD that involves a subset of {*A,C* }.

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A Geometric View of FD's

- Imagine the set of all instances of a particular relation.
- That is, all finite sets of tuples that have the proper number of components.
- Each instance is a point in this space.

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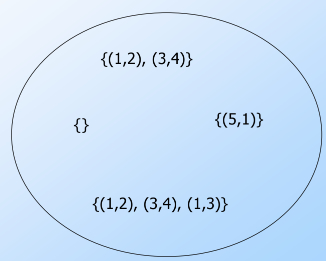
Example: R(A,B)



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An FD is a Subset of Instances

- For each FD *X* -> *A* there is a subset of all instances that satisfy the FD.
- We can represent an FD by a region in the space.
- *Trivial FD*: an FD that is represented by the entire space.
  - Example: *A* -> *A*.

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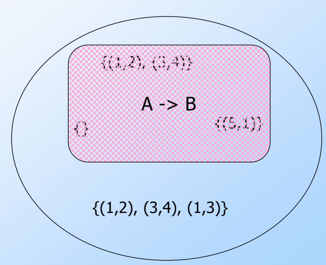
Example: A -> B for R(A,B)



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Representing Sets of FD's

- If each FD is a set of relation instances, then a collection of FD's corresponds to the intersection of those sets.
  - Intersection = all instances that satisfy all of the FD's.

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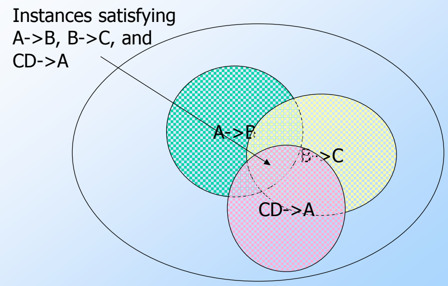
Example



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Implication of FD's

- If an FD Y -> B follows from FD's X_1 -> A_1,…, X_n -> A_n, then the region in the space of instances for Y -> B must include the intersection of the regions for the FD's X_i -> A_i.
  - That is, every instance satisfying all the FD's X_i -> A_i surely satisfies Y -> B.
  - But an instance could satisfy Y -> B, yet not be in this intersection.

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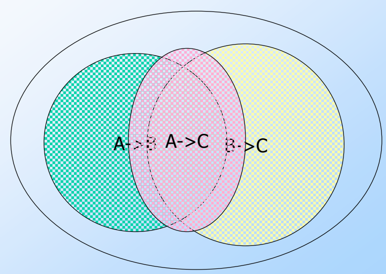
Example



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