# Multivalued Dependencies

Fourth Normal Form

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## A New Form of Redundancy

- Multivalued dependencies (MVD's) express a condition among tuples of a relation that exists when the relation is trying to represent more than one many-many relationship.
- Then certain attributes become independent of one another, and their values must appear in all combinations.

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## Example

Drinkers(name, addr, phones, beersLiked)

- A drinker's phones are independent of the beers they like.
- Thus, each of a drinker's phones appears with each of the beers they like in all combinations.
- This repetition is unlike redundancy due to FD's, of which name->addr is the only one.

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## Tuples Implied by Independence

Given **name, addr ->-> phones** is operative if we have the tuples *above* the dashed line:

| name | addr | phones | beersLiked |
|---|---|---|---|
| sue | a | p1 | b1 |
| sue | a | p2 | b2 |
| sue | a | p2 | b1 |
| sue | a | p1 | b2 |

then the tuples *below* the dashed line must also be in the relation.

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## Definition of MVD

- A *multivalued dependency* (MVD) $X$ ->-> $Y$ is an assertion that if two tuples of a relation agree on all the attributes of $X$, then their components in the set of attributes $Y$ may be swapped, and the result will be two tuples that are also in the relation.

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## Example

- The name-addr-phones-beersLiked example illustrated the MVD

  name->->phones

  and the MVD

  name ->-> beersLiked.

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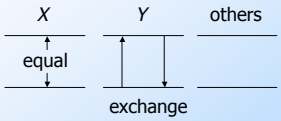
## Picture of MVD *X* ->-> *Y*

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## MVD Rules

- Every FD is an MVD.
  - If *X* -> *Y*, then swapping *Y*'s between two tuples that agree on *X* doesn't change the tuples.
  - Therefore, the "new" tuples are surely in the relation, and we know *X* ->-> *Y*.
- *Complementation* : If *X* ->-> *Y*, and *Z* is all the other attributes, then *X* ->-> *Z*.

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## Splitting Doesn't Hold

- Like FD's, we cannot generally split the left side of an MVD.
- But ***unlike*** FD's, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

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## Example

- Consider a drinkers relation:
  Drinkers(name, areaCode, phone, beersLiked, manf)
- A drinker can have several phones, with the number divided between areaCode and phone (last 7 digits).
- A drinker can like several beers, each with its own manufacturer.

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## Example, Continued

- Since the areaCode-phone combinations for a drinker are independent of the beersLiked-manf combinations, we expect that the following MVD's hold:
  name ->-> areaCode phone
  name ->-> beersLiked manf

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## Example Data

Here is possible data satisfying these MVD's:

| name | areaCode | phone | beersLiked | manf |
|---|---|---|---|---|
| Sue | 650 | 555-1111 | Bud | A.B. |
| Sue | 650 | 555-1111 | WickedAle | Pete's |
| Sue | 415 | 555-9999 | Bud | A.B. |
| Sue | 415 | 555-9999 | WickedAle | Pete's |

But we cannot swap area codes or phones my themselves. That is, neither name ->-> areaCode nor name ->-> phone holds for this relation.

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## Fourth Normal Form

- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.
- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to *decomposition*, but not when determining keys of the relation.

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## 4NF Definition

- A relation $R$ is in 4NF if whenever $X \twoheadrightarrow Y$ is a nontrivial MVD, then $X$ is a superkey.
  - "Nontrivial means that:
    1. $Y$ is not a subset of $X$, and
    2. $X$ and $Y$ are not, together, all the attributes.
  - Note that the definition of "superkey" still depends on FD's only.

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## BCNF Versus 4NF

- Remember that every FD $X \rightarrow Y$ is also an MVD, $X \twoheadrightarrow Y$.
- Thus, if $R$ is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation.
- But $R$ could be in BCNF and not 4NF, because MVD's are "invisible" to BCNF.

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## Decomposition and 4NF

- If $X \twoheadrightarrow Y$ is a 4NF violation for relation $R$, we can decompose $R$ using the same technique as for BCNF.
  1. $XY$ is one of the decomposed relations.
  2. All but $Y - X$ is the other.

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## Example

Drinkers(<u>name</u>, addr, <u>phones</u>, <u>beersLiked</u>)

FD: name -> addr
MVD's: name ->-> phones
name ->-> beersLiked

- Key is {name, phones, beersLiked}.
- All dependencies violate 4NF.

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## Example, Continued

- Decompose using name -> addr:

1. Drinkers1(<u>name</u>, addr)
   - In 4NF, only dependency is name -> addr.
2. Drinkers2(<u>name</u>, <u>phones</u>, <u>beersLiked</u>)
   - Not in 4NF. MVD's name ->-> phones and name ->-> beersLiked apply. No FD's, so all three attributes form the key.

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## Example: Decompose Drinkers2

- Either MVD name ->-> phones or name ->-> beersLiked tells us to decompose to:
  - Drinkers3(name, phones)
  - Drinkers4(name, beersLiked)

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