Multivalued Dependencies

Fourth Normal Form
A New Form of Redundancy

- Multivalued dependencies (MVD’s) express a condition among tuples of a relation that exists when the relation is trying to represent more than one many-many relationship.
- Then certain attributes become independent of one another, and their values must appear in all combinations.
Example

Drinkers(name, addr, phones, beersLiked)

- A drinker’s phones are independent of the beers they like.
- Thus, each of a drinker’s phones appears with each of the beers they like in all combinations.
- This repetition is unlike redundancy due to FD’s, of which name->addr is the only one.
## Tuples Implied by Independence

Given **name, addr -/>-> phones** is operative if we have the tuples *above* the dashed line:

<table>
<thead>
<tr>
<th>name</th>
<th>addr</th>
<th>phones</th>
<th>beersLiked</th>
</tr>
</thead>
<tbody>
<tr>
<td>sue</td>
<td>a</td>
<td>p1</td>
<td>b1</td>
</tr>
<tr>
<td>sue</td>
<td>a</td>
<td>p2</td>
<td>b2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sue</td>
<td>a</td>
<td>p2</td>
<td>b1</td>
</tr>
<tr>
<td>sue</td>
<td>a</td>
<td>p1</td>
<td>b2</td>
</tr>
</tbody>
</table>

then the tuples *below* the dashed line must also be in the relation.
Definition of MVD

A multivalued dependency (MVD) $X \rightarrow\rightarrow Y$ is an assertion that if two tuples of a relation agree on all the attributes of $X$, then their components in the set of attributes $Y$ may be swapped, and the result will be two tuples that are also in the relation.
Example

The name-addr-phones-beersLiked example illustrated the MVD name->->phones and the MVD name ->- beersLiked.
Picture of MVD $X \rightarrow\rightarrow Y$

\[ X \quad \text{equal} \quad Y \quad \text{exchange} \quad \text{others} \]
MVD Rules

- Every FD is an MVD.
  - If $X \rightarrow Y$, then swapping $Y$’s between two tuples that agree on $X$ doesn’t change the tuples.
  - Therefore, the “new” tuples are surely in the relation, and we know $X \rightarrow\rightarrow Y$.

- Complementation: If $X \rightarrow\rightarrow Y$, and $Z$ is all the other attributes, then $X \rightarrow\rightarrow Z$. 
Splitting Doesn’t Hold

Like FD’s, we cannot generally split the left side of an MVD.

But *unlike* FD’s, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.
Example

◆ Consider a drinkers relation:
  Drinkers(name, areaCode, phone, beersLiked, manf)
◆ A drinker can have several phones, with
  the number divided between areaCode
  and phone (last 7 digits).
◆ A drinker can like several beers, each
  with its own manufacturer.
Example, Continued

Since the areaCode-phone combinations for a drinker are independent of the beersLiked-manf combinations, we expect that the following MVD’s hold:

- name --> areaCode phone
- name --> beersLiked manf
Example Data

Here is possible data satisfying these MVD’s:

<table>
<thead>
<tr>
<th>name</th>
<th>areaCode</th>
<th>phone</th>
<th>beersLiked</th>
<th>manf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>650</td>
<td>555-1111</td>
<td>Bud</td>
<td>A.B.</td>
</tr>
<tr>
<td>Sue</td>
<td>650</td>
<td>555-1111</td>
<td>WickedAle</td>
<td>Pete’s</td>
</tr>
<tr>
<td>Sue</td>
<td>415</td>
<td>555-9999</td>
<td>Bud</td>
<td>A.B.</td>
</tr>
<tr>
<td>Sue</td>
<td>415</td>
<td>555-9999</td>
<td>WickedAle</td>
<td>Pete’s</td>
</tr>
</tbody>
</table>

But we cannot swap area codes or phones my themselves. That is, neither name -→ areaCode nor name -→ phone holds for this relation.
Fourth Normal Form

- The redundancy that comes from MVD’s is not removable by putting the database schema in BCNF.

- There is a stronger normal form, called 4NF, that (intuitively) treats MVD’s as FD’s when it comes to decomposition, but not when determining keys of the relation.
4NF Definition

A relation $R$ is in 4NF if whenever $X \rightarrow\rightarrow Y$ is a nontrivial MVD, then $X$ is a superkey.

“Nontrivial means that:
1. $Y$ is not a subset of $X$, and
2. $X$ and $Y$ are not, together, all the attributes.

Note that the definition of “superkey” still depends on FD’s only.
BCNF Versus 4NF

- Remember that every FD $X \rightarrow Y$ is also an MVD, $X \rightarrow\rightarrow Y$.
- Thus, if $R$ is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation.
- But $R$ could be in BCNF and not 4NF, because MVD’s are “invisible” to BCNF.
Decomposition and 4NF

If $X \rightarrow\rightarrow Y$ is a 4NF violation for relation $R$, we can decompose $R$ using the same technique as for BCNF.

1. $XY$ is one of the decomposed relations.
2. All but $Y - X$ is the other.
Example

Drinkers(name, addr, phones, beersLiked)

FD: name -> addr
MVD’s: name -->-> phones
      name -->-> beersLiked

◆ Key is {name, phones, beersLiked}.
◆ All dependencies violate 4NF.
Example, Continued

- Decompose using name -> addr:
  1. Drinkers1(name, addr)
     - In 4NF, only dependency is name -> addr.
  2. Drinkers2(name, phones, beersLiked)
     - Not in 4NF. MVD’s name -›› phones and name -›› beersLiked apply. No FD’s, so all three attributes form the key.
Example: Decompose Drinkers2

Either MVD name \(\rightarrow\rightarrow\) phones or name \(\rightarrow\rightarrow\) beersLiked tells us to decompose to:

- Drinkers3(name, phones)
- Drinkers4(name, beersLiked)