Algorithms
Computer Science 140 & Mathematics 168
Instructor: B. Thom
Spring 2004
Homework 1a
Due on 1/22/04 (beginning of class)

1. [**15 points**] **Properties of Logs.**

   Let's begin by proving that $\log_b xy = \log_b x + \log_b y$. The proof goes as follows: Let $k = \log_b xy$, $\ell = \log_b x$, and $m = \log_b y$. Then,$b^k = xy$, $b^\ell = x$, and $b^m = y$, by the definition of the logarithm. Thus, $b^k = b^\ell b^m = b^{\ell+m}$ by properties of exponents. Thus, $k = \ell + m$, which is what we had set out to prove.

   Now give proofs for each of the following properties of logarithms. Write your proofs out carefully. You should assume that a, b, c, n are positive *real numbers* (not necessarily integers).

   (a) $\log_b a^n = n \log_b a$.
   (b) $\log_b a = \frac{\log_c a}{\log_c b}$.
   (c) $a^{\log_b n} = n^{\log_b a}$.

2. [**10 points**] **Growth of Functions.** In this problem you will be doing two proofs by induction. Please write your proofs clearly, with a basis, induction hypothesis, and induction step.

   (a) Use induction to show that $n^2 \geq 2n + 1$ for $n \geq 3$.
   (b) Now use induction to show that $2^n \geq n^2$ for $n \geq 5$. (Using the result from part (a) of this problem will be most useful in doing this proof correctly!)