CS 141, Advanced Topics in Algorithms
Spring 2004
Homework 2a
Due Wednesday, January 28

1. **[35 Points] Another Version of Discrete Calculus!** In this problem we explore another version of discrete calculus that is similar, but not identical, to the version we examined in class.

   (a) **[5 Points] Warmin' Up!** In class, we defined $\Delta(f(x))$ to be $f(x+1)-f(x)$. Here, we'll explore a new discrete derivative Δ' defined by $\Delta'(f(x)) = f(x) - f(x-1)$.

      i. What is $\Delta'(x^2)$?

      ii. What is $\Delta'(x^{\underline{2}})$?

   (b) **[5 Points] Yield to the Rising Power!** Aha! That didn't turn out so great. Let's apply the standard trick of "defining our way out of trouble!" In particular, we'll define yet another type of exponentiation. Let $x^{\overline{m}}$, pronounced "x to the m rising", be defined by $x(x+1)(x+2)\ldots(x+m-1)$. (Notice that this is the product of m consecutive terms.) What is $\Delta'(x^{\overline{m}})$? Show your work.

   (c) **[5 Points] Checking out the Properties of Δ'.**

      i. Use the definition of Δ' to prove that $\Delta'(f(x)+h(x)) = \Delta'(f(x)) + \Delta'(h(x))$.

      ii. Show that $\Delta'(c \cdot f(x)) = c \cdot \Delta'(f(x))$.

   (d) **[5 Points] And now for the New Definite Summation!** Now we are compelled to define $\sum_a^b{}' f(x)\delta x$ to be $g(b) - g(a)$ where $\Delta'(g(x)) = f(x)$. Prove the Second Fundamental Theorem of Discrete Calculus:

$$\sum_a^b{}' f(x)\delta x = \sum_{k=a+1}^{b} f(k).$$

   (e) **[5 Points] And now Some Applications...** Use the Second Fundamental Theorem of Discrete Calculus to find a closed form for the summation:

$$\sum_{k=1}^{n} k^2.$$

   (f) **[5 Points] Summation by Parts!** Now expand $\Delta'(u(x)v(x))$ and derive a new summation by parts formula from it.

   (g) **[5 Points] Using Summation by Parts.** So isn't this slick?! Now, use the Summation by Parts formula above to find a nice closed form for

$$\sum_{k=1}^{n} k2^k.$$

   Be sure to show each step of your work.