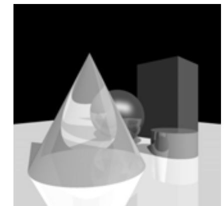
cs155 – z sweedyk

ray tracing

ray tracing

```
<scene>
  <cone material="glass">
  <sphere color="red">
  <box color="purple">
  <floor material="marble">
</scene>
```

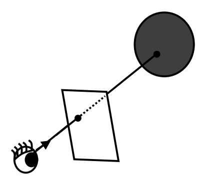


peter henry

ray tracing

- ray casting
  - rays
  - intersection tests
  - intersection with scene graph
  - lighting and material properties
- recursive ray tracing
- cheap tricks
- optimizations

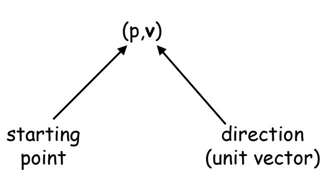
ray casting



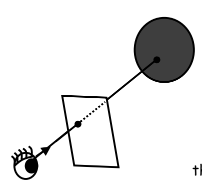
1. cast ray through pixel into scene
2. find intersection point (if any) that is closest to eye
3. compute color at intersection

ray specification

a ray is a half-line defined by a point and a unit vector



ray casting



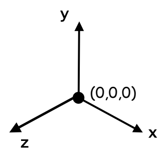
1. cast ray through pixel into scene

Construct the ray that starts at the viewpoint and goes through pixel i,j.

Where is the viewer?

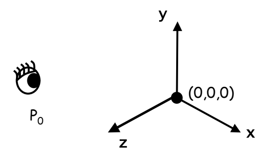
world coordinates

in the beginning there was (0,0,0) and x, y, z



Where is the viewer?

then "eye" came along

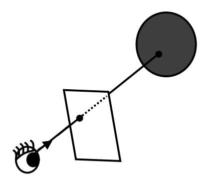


ray tracer input

- Viewpoint position P_0

⋮

ray casting



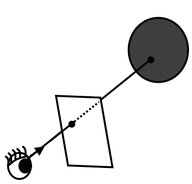
1. cast ray through pixel into scene

Construct the ray that starts at the viewer and goes through pixel i,j.

(P_0, ___)

Where is pixel i,j?

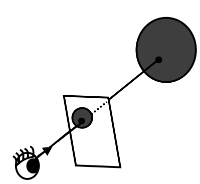
where is pixel i,j?



1. cast ray through pixel into scene

We imagine a *view window* suspended in our 3D world.

where is pixel i,j?



1. cast ray through pixel into scene

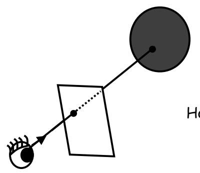
We imagine a *view window* suspended in our 3D world.

Our output is the projection of the scene onto this *view window*.

How do specify this *view window*?

where is pixel i,j?

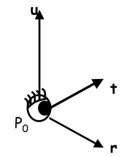
1. cast ray through pixel into scene



How is the viewer oriented in our 3D world?

How is viewer oriented?

3d world coordinates



- P_o is the viewpoint
- **t**, u and **r**=**t**×**u** are orthogonal unit vectors in the toward, up, and right directions

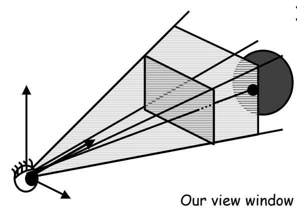
ray tracer input

- Viewpoint position P_0 and orientation **t**, **u**

⋮

where is pixel i,j?

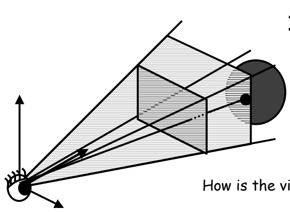
1. cast ray through pixel into scene



Our view window lies in a plane orthogonal to **t**.

where is pixel i,j?

1. cast ray through pixel into scene



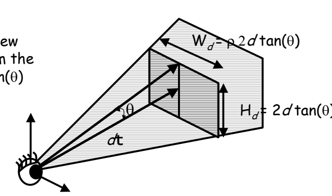
How is the viewer oriented in our 3D world?

How much of the world does the viewer see?

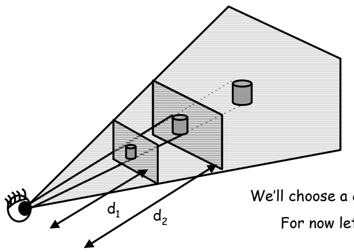
How much of the world does viewer see?

The view volume is specified by the half-height angle θ and the aspect ratio ρ=w/h.

The cross section of the view volume at a distance *d* from the viewer has height $H_d=2d\tan(\theta)$ and width $W_d = \rho H_d = \rho\, 2d \tan(\theta)$.



Which cross section is our view window??



We'll choose a convenient distance!

For now let's just call it *d*.

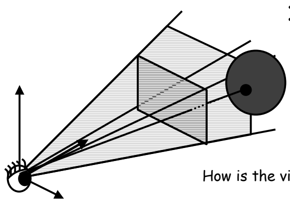
ray tracer input

- Viewpoint position P_0 and orientation **t, u**
- Half-height angle θ
- Aspect ratio ρ
- Distance to view window d

⋮

where is pixel i,j?

1. cast ray through pixel into scene



How is the viewer oriented in our 3D world?

How much of the world does the viewer see?

How densely should we sample the view window?

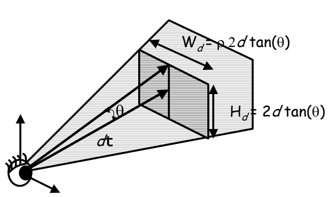
ray tracer input

- Viewpoint position P_0 and orientation **t, u**
- Half-height angle θ
- Aspect ratio ρ
- Distance to view window d
- Image width *w* and height *h* in pixels

⋮

w,h and the view volume

We require uniformly spaced pixels so $\rho = w/h$!

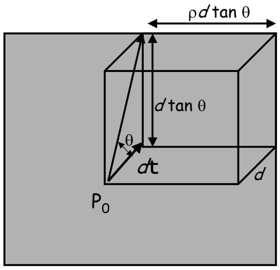


ray tracer input

- Viewpoint position P_0 and orientation **t, u**
- Half-height angle θ
- ~~Aspect ratio ρ~~
- Distance to view window d
- Image width *w* and height *h* in pixels

⋮

where is pixel i,j



What are the coordinates of the center of the view window?

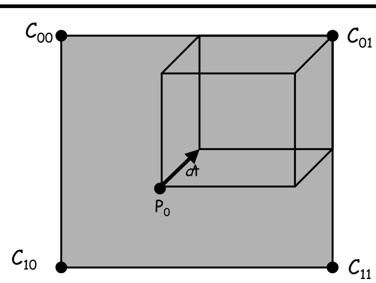
Ans: $P_0 + d\mathbf{t}$

point + vector

p + **v** is the point you get to by walking along the vector **v** from point p.

$q = p + v = (p_x+v_x, p_y+v_y, p_z+v_z)$

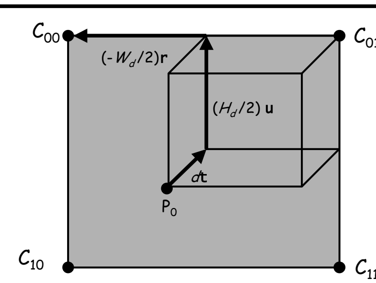
corner coordinates



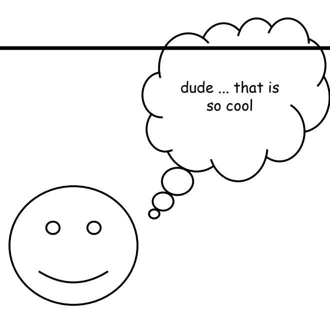
What are the coordinates of C_{00}?

Hint: Use **t**, **u**, **r**, H_d, W_d

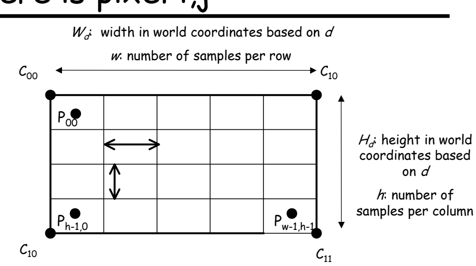
corner coordinates



$C_{00} = P_0 + d\mathbf{t} + (H_d/2)\,\mathbf{u} - (W_d/2)\,\mathbf{r}$

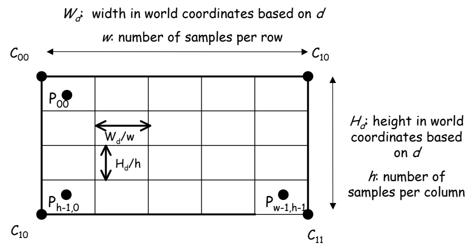


where is pixel i,j



What are the height and width of a grid cell in world coordinates?

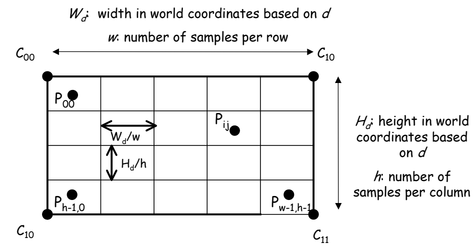
where is pixel i,j



What are the height and width of a grid cell in world coordinates?

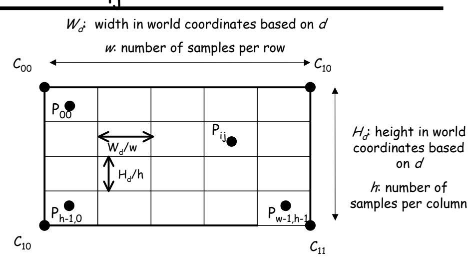
Ans: H_d/h and W_d/w

where is P_{ij}?



Hint: How do we get to P_{ij} from C_{00}?

where is P_{ij}?



$$P_{ij} = C_{00} + (i+\tfrac{1}{2})(W_d/w)\ \mathbf{r} - (j+\tfrac{1}{2})(H_d/h)\ \mathbf{u}$$

choosing d

$$P_{ij} = C_{00} + (i+\tfrac{1}{2})(W_d/w)\ \mathbf{r} - (j+\tfrac{1}{2})(H_d/h)\ \mathbf{u}$$

Let's choose d so that $H_d = h$.

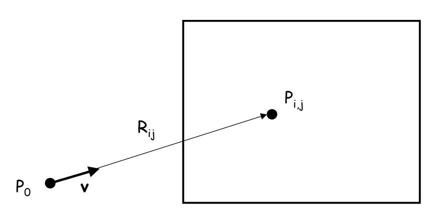
So $d = h/(2\tan(\theta))$.

Then $W_d = \rho H = \rho h = w$.

specification

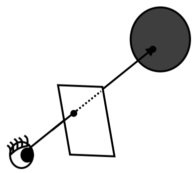
- Input specification
  - Viewpoint position P_0 and orientation **t**, **u**
  - Half-height angle θ
  - Image width w and height h in pixels
- Compute
  - Right vector $\mathbf{r} = \mathbf{t} \times \mathbf{u}$
  - Aspect ratio $\rho = w/h$
  - $d = h/(2\tan\theta)$
  - $H_d = h$, $W_d = w$

casting rays



$R_{ij} = (P_0, \mathbf{v_{ij}})$ where $\mathbf{v_{ij}} = (P_{i,j} - P_0)/|| P_{i,j} - P_0||$

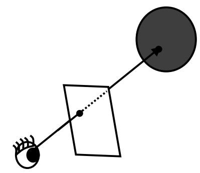
ray casting



- cast ray through pixel into scene

TA DA!

ray casting



- cast ray through pixel into scene
- **find intersection point (if any) that is closest to eye**
- compute color at intersection

intersection

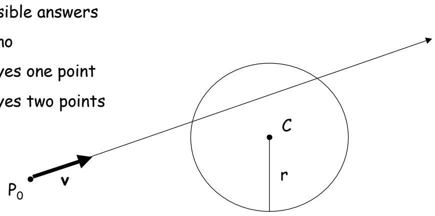
- sphere
- triangle
- box
- cylinder
- cone
- torus

intersection

- **sphere**
- triangle
- box
- cylinder
- cone
- torus

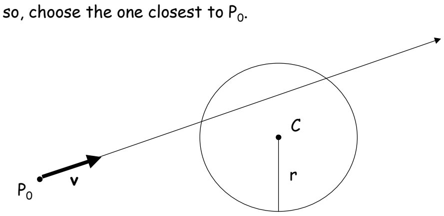
sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- Possible answers
  - no
  - yes one point
  - yes two points



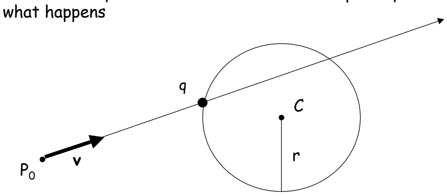
sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- If so, choose the one closest to P_0.



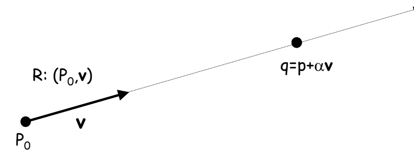
sphere intersection

- Is there a point that lies on the sphere and on the ray $(P_0,\mathbf{v})$?
- To answer this question we'll assume there is a point q and see what happens



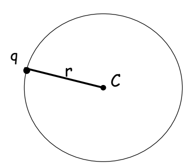
ray: parametric form

Since q lies on R=(p,**v**), q=p+α**v** for some $\alpha \geq 0$



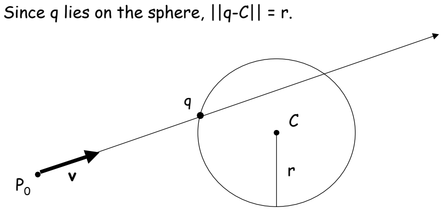
sphere intersection

Since q lies on the sphere, $||q-C|| = r$.



sphere intersection

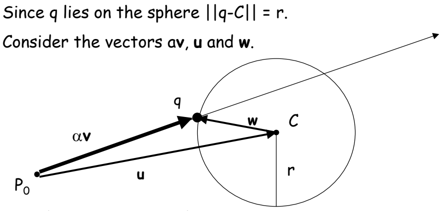
- Since q lies on R, there is some $\alpha \geq 0$ such that $q = P_0 + \alpha\mathbf{v}$.
- Since q lies on the sphere, $||q-C|| = r$.



Let's compute α!

sphere intersection

- Since q lies on R, there is some $\alpha \geq 0$ such that $q = P_0 + \alpha\mathbf{v}$.
- Since q lies on the sphere $||q-C|| = r$.
- Consider the vectors a**v**, **u** and **w**.



Then $\mathbf{w} = \alpha\mathbf{v} - \mathbf{u}$ and

$$r^2 = ||w||^2 = w \bullet w = (\alpha\mathbf{v} - \mathbf{u}) \bullet (\alpha\mathbf{v} - \mathbf{u}) = \alpha^2\,||\mathbf{v}||^2 - 2\alpha(\mathbf{u} \bullet \mathbf{v}) + ||\mathbf{u}||^2$$

sphere intersection

Does the ray intersect the sphere?

$\Updownarrow$

Does the quadratic (in α)

$$||\mathbf{v}||^2\,\alpha^2 - 2\,(\mathbf{u} \bullet \mathbf{v})\,\alpha + ||\mathbf{u}||^2 - r^2$$

have any real, non-negative roots?

- No: no intersection
- Yes: Intersection point at $q = P_0 + \beta\mathbf{v}$ where β is the smallest such root.

sphere intersection

Does the ray intersect the sphere?

⇕

Does the quadratic (in α)

$$||\mathbf{v}||^2 \alpha^2 - 2(\mathbf{u} \bullet \mathbf{v})\alpha + ||\mathbf{u}||^2 - r^2$$

have any real, non-negative roots?

What does it mean if there are no real roots?

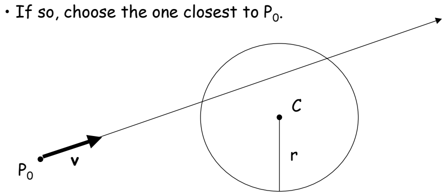
What does it mean if there is one real root?

What does it mean if there are two real roots?

What does it mean if a root is negative?

sphere intersection

- Is there a point that lies on the sphere and on the ray (P_0,**v**)?
- If so, choose the one closest to P_0.

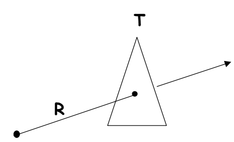


The closest intersection point is $P_0 + \beta\mathbf{v}$ where β is the smallest real non-negative root of $||\mathbf{v}||^2 \alpha^2 - 2(\mathbf{u} \bullet \mathbf{v})\alpha + ||\mathbf{u}||^2 - r^2$. (If there are no real non-negative roots then there is no intersection.)

intersection

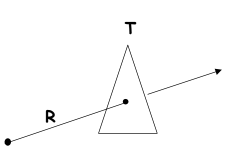
- sphere
- **triangle**
- box
- cylinder
- cone
- torus

triangle intersection



- do R and T intersect?
- if so, where?

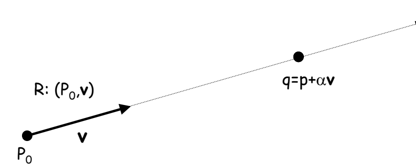
triangle intersection



1. Find the intersection point (if any) of R and the plane containing T.
2. Determine if the intersection point is inside the triangle.

ray: parametric form

a point q lies on R=(p,**v**) iff q=p+α**v** for some $\alpha \geq 0$



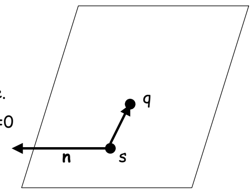
plane

Let **n** be a normal to the plane.

Let s be a point on the plane.

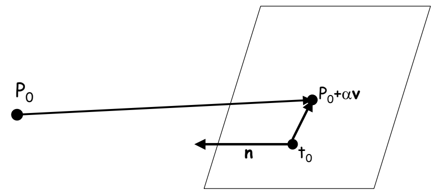
Let q be an arbitrary point in space.

Then q lies on the plane iff **n•**(q-s)=0



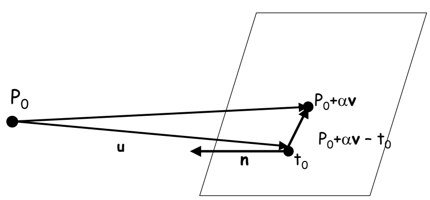
plane intersection

Is there a point $P_0+\alpha\mathbf{v}$, $a\geq 0$, such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$?



plane intersection

Is there a point $P_0+\alpha\mathbf{v}$, $a\geq 0$, such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$?



Assume there is and solve for α.

Then $0 = \mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = \mathbf{n}\bullet(\alpha\mathbf{v} - \mathbf{u})$ where **u** is the vector from P_0 to t_0.

plane intersection

Is there a point $P_0+\alpha\mathbf{v}$, $a\geq 0$, such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$?

⇕

Is there an α such that $\mathbf{n}\bullet(\alpha\mathbf{v} - \mathbf{u})=0$?

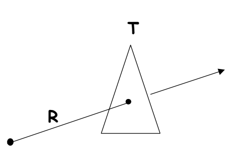
If $\mathbf{n}\bullet\mathbf{v}=0$ then R is either parallel to plane or it lies in the plane.

If $\mathbf{n}\bullet\mathbf{v}\neq 0$ then $\alpha = (\mathbf{n}\bullet\mathbf{u})/(\mathbf{n}\bullet\mathbf{v})$.

If $\alpha\geq 0$ then R intersects the plane at point $P_0+\alpha\mathbf{v}$.

If $\alpha<0$ then R does not intersect plane.

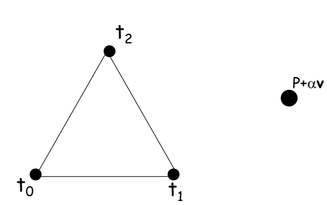
triangle intersection



1. Find the intersection point (if any) of R and the plane containing T.
2. Determine whether the intersection point lies inside the triangle.

triangle intersection

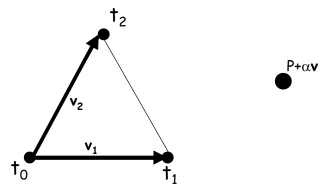
$P_0 + \alpha\mathbf{v}$ lies on the plane containing the triangle T



triangle plane

$P_0 + \alpha\mathbf{v}$ lies on the plane containing the triangle T

Let's define some more vectors!



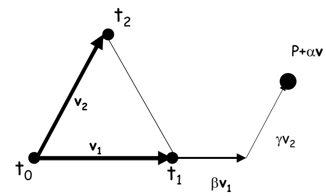
$v_1 = (t_1 - t_0)$

$v_2 = (t_2 - t_0)$

triangle: barycentric coordinates

Since $P_0 + \alpha\mathbf{v}$ lies on the plane containing the triangle T,

$P_0 + \alpha\mathbf{v} = t_0 + \beta\mathbf{v_1} + \gamma\mathbf{v_2}$ for some β and γ.



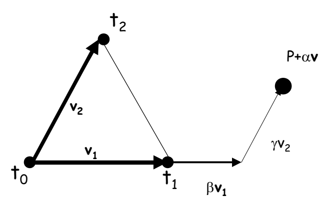
$v_1 = (t_1 - t_0)$

$v_2 = (t_2 - t_0)$

triangle: barycentric coordinates

Since $P_0 + \alpha\mathbf{v}$ lies on the plane containing the triangle T,

$P_0 + \alpha\mathbf{v} = t_0 + \beta\mathbf{v_1} + \gamma\mathbf{v_2}$ for some β and γ.

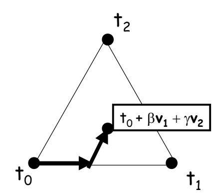


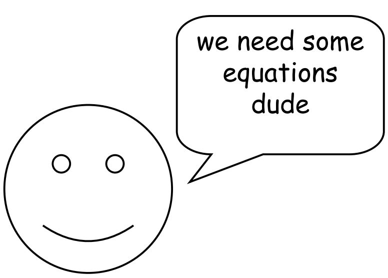
$P + \alpha\mathbf{v}$ lies on the triangle iff β and γ satisfy ________________

triangle: barycentric coordinates

$P + \alpha\mathbf{v}$ lies on the triangle T iff $P + \alpha\mathbf{v} = t_0 + \beta\mathbf{u} + \gamma\mathbf{w}$ where

$\beta \geq 0$, $\gamma \geq 0$, $\beta + \gamma \leq 1$





triangle intersection

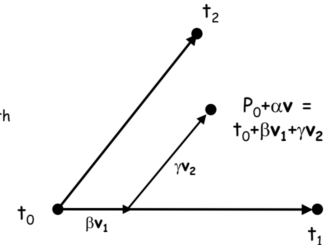
$P_0 + \alpha\mathbf{v} = t_0 + \beta\mathbf{v_1} + \gamma\mathbf{v_2}$

This is a vector equations with unknowns β and γ.

You can do this!!

But why don't I help.

(Warning: algebra ahead!)

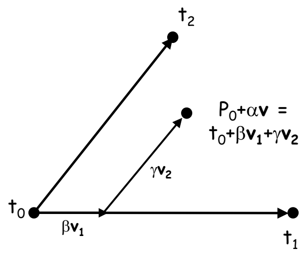


triangle intersection

$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$P_0+\alpha\mathbf{v} - t_0 = \beta\mathbf{v_1}+\gamma\mathbf{v_2}$

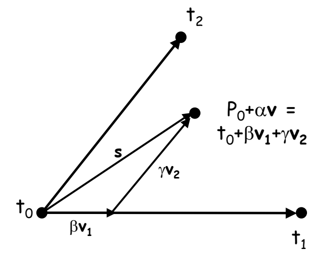


triangle intersection

$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$\underbrace{P_0+\alpha\mathbf{v} - t_0}_{\mathbf{s}} = \beta\mathbf{v_1}+\gamma\mathbf{v_2}$



triangle intersection

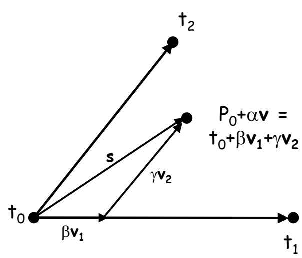
$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$\underbrace{P_0+\alpha\mathbf{v} - t_0}_{\mathbf{s}} = \beta\mathbf{v_1}+\gamma\mathbf{v_2}$

Remember $t_0=P_0 + \mathbf{u}$

so $\mathbf{s} = \alpha\mathbf{v} - \mathbf{u}$.



triangle intersection

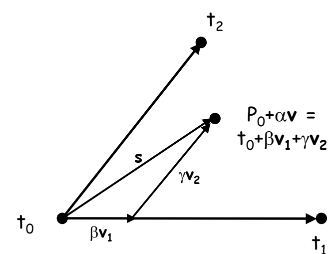
$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$P_0+\alpha\mathbf{v} - t_0 = \beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$\underbrace{\alpha\mathbf{v} - \mathbf{u}}_{\mathbf{s}} = \beta\mathbf{v_1} + \gamma\,\mathbf{v_2}$



triangle intersection

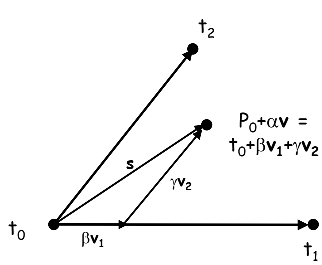
$P_0+\alpha\mathbf{v} = t_0+\beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$P_0+\alpha\mathbf{v} - t_0 = \beta\mathbf{v_1}+\gamma\mathbf{v_2}$

$\Updownarrow$

$\underbrace{\alpha\mathbf{v} - \mathbf{u}}_{\mathbf{s}} = \beta\mathbf{v_1} + \gamma\,\mathbf{v_2}$



2 Equations $\mathbf{s} \bullet \mathbf{v_1} = \beta(\mathbf{v_1} \bullet \mathbf{v_1}) + \gamma(\mathbf{v_1} \bullet \mathbf{v_2})$

2 Unknowns $\mathbf{s} \bullet \mathbf{v_2} = \beta(\mathbf{v_1} \bullet \mathbf{v_2}) + \gamma(\mathbf{v_2} \bullet \mathbf{v_2})$

recap: triangle intersection

- Find α such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$.

  (If no solutions: no intersection.
  If infinitely many solutions then ray lies in plane.)

recap: triangle intersection

- Find α such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$.
- Compute $v_1=t_1-t_0$, $v_2=t_2-t_0$, $u=t_0-P_0$, and $s=\alpha v-u$.

recap: triangle intersection

- Find α such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$.
- Compute $v_1=t_1-t_0$, $v_2=t_2-t_0$, $u=t_0-P_0$, and $s=\alpha v-u$.
- Solve system of equations:

$$\mathbf{s} \bullet \mathbf{v}_1 = \beta(\mathbf{v}_1 \bullet \mathbf{v}_1) + \gamma(\mathbf{v}_1 \bullet \mathbf{v}_2)$$
$$\mathbf{s} \bullet \mathbf{v}_2 = \beta(\mathbf{v}_1 \bullet \mathbf{v}_2) + \gamma(\mathbf{v}_2 \bullet \mathbf{v}_2)$$

for β and γ.

There is a unique solution provided T really is a triangle!

triangle intersection

- Find α such that $\mathbf{n}\bullet(P_0+\alpha\mathbf{v} - t_0) = 0$.
- Compute $v_1=t_1-t_0$, $v_2=t_2-t_0$, $u=t_0-P_0$, and $s=\alpha v-u$.
- Solve system of equations:

$$\mathbf{s} \bullet \mathbf{v}_1 = \beta(\mathbf{v}_1 \bullet \mathbf{v}_1) + \gamma(\mathbf{v}_1 \bullet \mathbf{v}_2)$$
$$\mathbf{s} \bullet \mathbf{v}_2 = \beta(\mathbf{v}_1 \bullet \mathbf{v}_2) + \gamma(\mathbf{v}_2 \bullet \mathbf{v}_2)$$

  for β and γ.
- If $\beta\geq 0$, $\gamma\geq 0$, $\beta+\gamma\leq 1$ then the ray intersects the triangle at $P_0+\alpha\mathbf{v}$. Else there is no intersection.

intersection

- triangle
- sphere
- box
- cylinder
- cone
- torus

Etc. – you get to do these if you so choose!