Assignment 1, Due Mon., 24 Jan.

Construct natural deduction proofs for the following sequents:

1. $\vdash p \rightarrow (q \rightarrow p)$
2. $p \rightarrow (q \vee r) \vdash (p \rightarrow q) \vee (p \rightarrow r)$
3. $(\neg p \wedge \neg q) \vdash \neg(p \vee q)$
4. $\neg(p \vee q) \vdash (\neg p \wedge \neg q)$
5. $\vdash (\neg p \rightarrow p) \rightarrow p$

Solutions in the suggested order: 1,4,5,3,2

1. $\vdash p \rightarrow (q \rightarrow p)$

1.	p	Assumption
2.	q	Assumption
3.	p	copy 1
4.	$q \rightarrow p$	$\rightarrow$i 2-3
5.	$p \rightarrow (q \rightarrow p)$	$\rightarrow$i 1-4

4. $\neg(p \vee q) \vdash (\neg p \wedge \neg q)$

1.	$\neg(p \vee q)$	Premise
2.	p	Assumption
3.	$p \vee q$	$\vee i_1$ 2
4.	$\bot$	$\neg e$ 3, 1
5.	$\neg p$	$\neg i$ 2-4
6.	q	Assumption
7.	$p \vee q$	$\vee i_2$ 6
8.	$\bot$	$\neg e$ 1, 7
9.	$\neg q$	$\neg i$ 6-8
10.	$(\neg p \wedge \neg q)$	$\wedge i$ 5, 9

5. $\vdash (\neg p \rightarrow p) \rightarrow p$

1.	$(\neg p \rightarrow p)$	Assumption
2.	$\neg p$	Assumption
3.	p	$\rightarrow$e 2, 1
4.	$\bot$	$\neg$e 3, 2
5.	p	RAA 2-4
6.	$(\neg p \rightarrow p) \rightarrow p$	$\rightarrow$i 1-5

3. $(\neg p \wedge \neg q) \vdash \neg(p \vee q)$

1.	$\neg p \wedge \neg q$	Premise
2.	$\neg p$	$\wedge e_1$
3.	$\neg q$	$\wedge e_2$
4.	$p \vee q$	Assumption
5.	p	Assumption
6.	$\bot$	$\neg e$ 5, 2
7.	$\neg(p \vee q)$	$\bot e$ 6
8.	q	Assumption
9.	$\bot$	$\neg e$ 8, 3
10.	$\neg(p \vee q)$	$\bot e$ 9
11.	$\neg(p \vee q)$	$\vee e$ 4, 5-7, 8-10
12.	$\bot$	$\neg e$ 4, 11
13.	$\neg(p \vee q)$	$\neg i$ 4-12

2. 1st level structure of $p \rightarrow (q \vee r) \vdash (p \rightarrow q) \vee (p \rightarrow r)$

1.	$p \rightarrow (q \vee r)$	Premise
2.	$p \vee \neg p$	LEM
3.	p	Assumption
4.	.	
5.	.	
6.	.	
7.	.	
8.	.	
9.	.	
10.	.	
11.	.	
12.	.	
13.	.	
14.	.	
15.	$(p \rightarrow q) \vee (p \rightarrow r)$	
16.	$\neg p$	Assumption
17.	.	
18.	.	
19.	.	
20.	.	
21.	$(p \rightarrow q) \vee (p \rightarrow r)$	
22.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee$e 2, 3-15, 16-21

2. 2nd level structure of $p \rightarrow (q \vee r) \vdash (p \rightarrow q) \vee (p \rightarrow r)$

1.	$p \rightarrow (q \vee r)$	Premise
2.	$p \vee \neg p$	LEM
3.	p	Assumption
4.	$q \vee r$	$\rightarrow$e 1, 3
5.	q	Assumption
6.	.	
7.	.	
8.	.	
9.	$(p \rightarrow q) \vee (p \rightarrow r)$	
10.	r	Assumption
11.	.	
12.	.	
13.	.	
14.	$(p \rightarrow q) \vee (p \rightarrow r)$	
15.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee$e 4, 5-9, 10-14
16.	$\neg p$	Assumption
17.	.	
18.	.	
19.	.	
20.	.	
21.	$(p \rightarrow q) \vee (p \rightarrow r)$	
22.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee$e 2, 3-15, 16-21

2. $p \rightarrow (q \vee r) \vdash (p \rightarrow q) \vee (p \rightarrow r)$

1.	$p \rightarrow (q \vee r)$	Premise
2.	$p \vee \neg p$	LEM
3.	$\quad p$	Assumption
4.	$\quad q \vee r$	$\rightarrow$e 1, 3
5.	$\quad q$	Assumption
6.	$\quad\quad p$	Assumption
7.	$\quad\quad q$	copy 5
8.	$\quad p \rightarrow q$	$\rightarrow$i 6, 7
9.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee i_1$ 8
10.	$\quad r$	Assumption
11.	$\quad\quad p$	Assumption
12.	$\quad\quad r$	copy 10
13.	$\quad p \rightarrow r$	$\rightarrow$i 11-12
14.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee i_2$ 13
15.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee$e 4, 5-9, 10-14
16.	$\neg p$	Assumption
17.	$\quad p$	Assumption
18.	$\quad \bot$	$\neg$e 16, 17
19.	$\quad q$	$\bot$e 18
20.	$(p \rightarrow q)$	$\rightarrow$i 17-19
21.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee i_1$ 20
22.	$(p \rightarrow q) \vee (p \rightarrow r)$	$\vee$e 2, 3-15, 16-21

Possibly-Useful Derived Rules for the Future

- $\dfrac{\neg\varphi}{\varphi\rightarrow\psi}$
- $\dfrac{\psi}{\varphi\rightarrow\psi}$
- $\dfrac{\varphi \vee \psi \quad \neg\varphi}{\psi}$ (DS_1, Disjunctive Syllogism)
- $\dfrac{\varphi \vee \psi \quad \neg\psi}{\varphi}$ (DS_2, Disjunctive Syllogism)