CS 81 Assignment 3 for Wed., Feb. 9

1. Demonstrate how the construction in the proof of the completeness theorem ($\models \eta$ implies $\vdash \eta$) would construct a proof of the tautology

   $(p \rightarrow q) \rightarrow \neg(p \wedge \neg q)$.

2. Construct a natural deduction proof for these sequents:

   a. $(\forall x)\ p(x) \vee (\forall x)\ q(x) \ \vdash\ ((\forall x)\ p(x) \vee q(x))$

   b. $(\exists x)\ p(x) \vee (\exists x)\ q(x) \ \vdash\ ((\exists x)\ p(x) \vee q(x))$

3. Construct natural deduction proofs for two analogs of the DeMorgan rules:

   a. $(\forall x)\ p(x) \ \vdash\ \neg(\exists x)\ \neg p(x)$

   b. $(\exists x)\ p(x) \ \vdash\ \neg(\forall x)\ \neg p(x)$

4. Express the premises and conclusion below in predicate logic. Then prove the combination as a sequent:

   a. All computer scientists can program.
   b. Anyone who can program is proficient with logic.
   c. No one who is proficient with logic holds a public office.
   d. Therefore no one holding a public office is a computer scientist.

5. Convince yourself that the following assertions can each be true or false for a specific 2-ary predicate p on a non-empty domain:

   a. Reflexive: $(\forall x)\ p(x, x)$

   b. Symmetric: $(\forall x)\ (\forall y)\ (p(x, y) \rightarrow p(y, x))$

   c. Transitive: $(\forall x)\ (\forall y)\ (\forall z)\ (p(x, y) \wedge p(y, z) \rightarrow p(x, z))$

   The following English-language argument is given to justify that Transitive and Symmetric together imply Reflexive: “Let x_0 be an arbitrary element. Suppose that y_0 is such that $p(x_0, y_0)$. Then by the symmetric property, also $p(y_0, x_0)$. But then by the transitive property, from $p(x_0, y_0) \wedge p(y_0, x_0)$ we get $p(x_0, x_0)$. Since x_0 was arbitrary, it follows that $(\forall x)\ p(x, x)$, which is just the reflexive property.”

   If this argument is correct, show the proof using natural deduction. If it is incorrect, demonstrate why.