CS 81 Solutions to Assignment 6 for Wed., Mar. 23

Exercises in Hein, pages 530-534

1.b Transform to clausal form: $(A \wedge B) \vee (C \wedge D) \vee (E \rightarrow F)$:

1. Replace $\rightarrow$, giving $(A \wedge B) \vee (C \wedge D) \vee \neg E \vee F$.
2. Using the distributive law $((X \wedge Y) \vee Z) \equiv (X \vee Z) \wedge (Y \vee Z)$,

   distribute $((C \wedge D) \vee \neg E \vee F)$ over $(A \wedge B)$ to get

   $(A \vee (C \wedge D) \vee \neg E \vee F) \wedge (B \vee (C \wedge D) \vee \neg E \vee F)$.
3. Interchange the first two disjuncts in each conjunct, using the commutative law, to get

   $((C \wedge D) \vee A \vee \neg E \vee F) \wedge ((C \wedge D) \vee B \vee \neg E \vee F)$.
4. Use the distributive law again, to get

   $(C \vee A \vee \neg E \vee F) \wedge (D \vee A \vee \neg E \vee F)$
   $\wedge (C \vee B \vee \neg E \vee F) \wedge (D \vee B \vee \neg E \vee F)$
5. (optional) Reorder the disjuncts, to get:

   $(A \vee C \vee \neg E \vee F) \wedge (A \vee D \vee \neg E \vee F)$
   $\wedge (B \vee C \vee \neg E \vee F) \wedge (B \vee D \vee \neg E \vee F)$

3.c Find a resolution proof of

$\{ A \vee B, A \vee \neg C, \neg A \vee C, \neg A \vee \neg B, C \vee \neg B, \neg C \vee B \}$

1.	$A \vee B$	Premise
2.	$A \vee \neg C$	Premise
3.	$\neg A \vee C$	Premise
4.	$\neg A \vee \neg B$	Premise
5.	$C \vee \neg B$	Premise
6.	$\neg C \vee B$	Premise
7.	$B \vee C$	Resolution 1, 3
8.	C	Resolution 5, 7
9.	B	Resolution 6, 8
10.	$\neg A$	Resolution 4, 9
11.	A	Resolution 2, 8
12.	$\perp$	Resolution 11, 12

4.e Compose the substitions $\theta = \{x/y, y/f(z)\}$ $\sigma = \{y/f(a), z/b\}$.

$\theta\sigma = \{x/f(a), y/f(b), z/b\}$.

6.c Use the unification algorithm to find an mgu for:

$\{p(f(x, g(y)), y), p(f(g(a), z), b)\}$

1. $S = \{[p(f(x, g(y)), y), p(f(g(a), z), b)]\}$
   $\mu = \{\}$

2. $S = \{[f(x, g(y)), f(g(a), z)], [y, b]\}$
   $\mu = \{\}$

3. $S = \{[g(y), z], [y, b]\}$
   $\mu = \{x/g(a)\}$

4. $S = \{[y, b]\}$
   $\mu = \{x/g(a), z/g(y)\}$

5. $S = \{\}$
   $\mu = \{x/g(a), z/g(b), y/b\}$ is the mgu.

8.c Use resolution to show unsatisfiable: $\{p(a) \vee p(x), \neg p(a) \vee \neg p(y)\}$

1.	$p(a) \vee p(x)$	Premise
2.	$\neg p(a) \vee \neg p(y)$	Premise
3.	$p(x)$	Resolution 1, 2 (with $\{y/a\}$)
4.	$\neg p(a)$	Resolution 2, 3 (with $\{x/y\}$)
5.	$\perp$	Resolution 3, 4 (with $\{x/a\}$)

8.e Use resolution to show unsatisfiable:
$\{q(x) \vee q(a), \neg p(y) \vee \neg p(g(a)) \vee \neg q(a), p(z) \vee p(g(w)) \vee \neg q(w)\}$

This problem seems to require simultaneous unification of several literals in a single clause (equivalent to factoring prior to resolving). I wasn't able to do it with simple binary resolution, although I won't swear it can't be done.

1.	$q(x) \vee q(a)$	Premise
2.	$\neg p(y) \vee \neg p(g(a)) \vee \neg q(a)$	Premise
3.	$p(z) \vee p(g(w)) \vee \neg q(w)$	Premise
4.	$\neg p(y) \vee \neg p(g(a))$	Resolution 1, 2 (with $\{x/a\}$)
5.	$\neg q(a)$	Resolution 3, 4 (with $\{w/a, z/g(a), y/g(a)\}$)
6.	$\perp$	Resolution 1, 5 (with $\{x/a\}$)

9.c Prove by resolution: $((p \vee q) \wedge (q \rightarrow r) \wedge (r \rightarrow s)) \rightarrow (p \vee s)$

Negate and convert to clauses:

1.	$p \vee q$	Premise
2.	$\neg q \vee r$	Premise
3.	$\neg r \vee s$	Premise
4.	$\neg p$	Premise
5.	$\neg s$	Premise
6.	q	Resolution 1, 4
7.	r	Resolution 2, 6
8.	s	Resolution 3, 7
9.	$\bot$	Resolution 5, 8

12.b Translate to first-order predicate calculus, then prove using resolution:

1. $\forall x\ (baby(x) \rightarrow illogical(x))$ “Babies are illogical.”
2. $\neg\exists x\ (despised(x) \wedge can\_manage(x))$ “Nobody is despised who can manage a crocodile.”
3. $\forall x\ (illogical(x) \rightarrow despised(x))$ “Illogical persons are despised.”
   Therefore:
4. $\forall x\ (baby(x) \rightarrow \neg can\_manage(x))$ “Babies cannot manage crocodiles.”

To get to clausal form, we retain the premises and negate the conclusion, also using DeMorgan’s law on the second statement:

1. $\forall x\ (baby(x) \rightarrow illogical(x))$ “Babies are illogical.”
2. $\forall x\ \neg(despised(x) \wedge can\_manage(x))$ “Nobody is despised who can manage a crocodile.”
3. $\forall x\ (illogical(x) \rightarrow despised(x))$ “Illogical persons are despised.”
   Therefore:
4. $\neg\forall x\ (baby(x) \rightarrow \neg can\_manage(x))$ “Not (babies cannot manage crocodiles).”

The fourth statement is equivalent to:

4’. $\exists x\ \neg(baby(x) \rightarrow \neg can\_manage(x))$

which is in turn equivalent to:

4’’. $\exists x\ (baby(x) \wedge can\_manage(x))$

We need to replace the $\exists$ quantified variable with a Skolem constant:

4’’’. $baby(b) \wedge can\_manage(b)$ “Baby b can manage crocodiles.”

The last statement translates to two separate clauses. Statements 1-3 translate to clauses by removing quantifiers, replacing → and using DeMorgan's law:

1.	¬baby(x) ∨ illogical(x)	Premise
2.	¬despised(x) ∨ ¬can_manage(x)	Premise
3.	¬illogical(x) ∨ despised(x)	Premise
4.	baby(b)	Denial of conclusion
5.	can_manage(b)	Denial of conclusion
6.	illogical(b)	Resolution 1, 4
7.	¬despised(b)	Resolution 2, 5
8.	¬illogical(b)	Resolution 3, 7
9.	⊥	Resolution 6, 8