CS 81 Assignment 9 for Wed., April 13

1. Show that the following functions are primitive-recursive:

   a. gt(x, y) = 1 if $x > y$; 0 otherwise
   b. lte(x, y) = 1 if $x \leq y$; 0 otherwise

2. Suppose that g is a primitive-recursive function of two arguments. Show that the following are also primitive recursive:

   a. $s(n, x) = g(0, x) + g(1, x) + \ldots. + g(n, x)$
   b. m(b, x) = the least $y \leq b$ such that $g(y, x) = 0$.

3. Show that the following functions are primitive-recursive:

   a. isPrime(x) = 1 if x has no divisors other than 1 and x; 0 otherwise
   b. prime(n) = the n^{th} prime number (p(0) = 2, p(1) = 3, p(2) = 5, …).

4. Suppose that L and M are recursive languages (meaning languages accepted by Turing machines). Classify whether the following are true or false, justifying each answer. (Note that if the answer is false, you only need to give a counterexample).

   a. $L \cup M$ is recursive.
   b. $L \cap M$ is recursive.
   c. L - M is recursive.

5. Suppose that L and M are recursively-enumerable languages (meaning languages recognized by Turing machines). Classify whether the following are true or false, justifying each answer. (Note that if the answer is false, you only need to give a counterexample).

   a. $L \cup M$ is recursively-enumerable.
   b. $L \cap M$ is recursively-enumerable.
   c. L - M is recursively-enumerable.

6. State whether or not there is a decision procedure for each of the following questions about a language $L \subseteq \{a, b\}^*$ as the language accepted by a Turing machine.

   a. $L = \emptyset$.
   b. $L = \{a, b\}^*$.
   c. $\{aba\} \subseteq L$.
   d. $L \subseteq \{aba\}$.
   e. $L \not\subseteq \{aba\}$.
   f. L is recursive.
   g. L is finite.