

Some proofs I constructed using the JAPE editor (see Links web page for downloads).

Note that the bottom two are essentially the converses of the top two.

However, the top two are intuitionistic, whereas the bottom two are not.

<p>1: $\forall x.R(x)$ premise</p> <p>2: $\exists y.\neg R(y)$ assumption</p> <p>3: actual i, $\neg R(i)$ assumptions</p> <p>4: $R(i)$ \forall elim 1,3.1</p> <p>5: \perp \neg elim 4,3.2</p> <p>6: \perp \exists elim 2,3-5</p> <p>7: $\neg\exists y.\neg R(y)$ \neg intro 2-6</p>	<p>1: $\exists x.R(x)$ premise</p> <p>2: $\forall y.\neg R(y)$ assumption</p> <p>3: actual i, $R(i)$ assumptions</p> <p>4: $\neg R(i)$ \forall elim 2,3.1</p> <p>5: \perp \neg elim 3.2,4</p> <p>6: \perp \exists elim 1,3-5</p> <p>7: $\neg\forall y.\neg R(y)$ \neg intro 2-6</p>
<p>1: $\neg\forall x.R(x)$ premise</p> <p>2: $\neg\exists y.\neg R(y)$ assumption</p> <p>3: actual i assumption</p> <p>4: $\neg R(i)$ assumption</p> <p>5: $\exists y.\neg R(y)$ \exists intro 4,3</p> <p>6: \perp \neg elim 5,2</p> <p>7: $R(i)$ contra (classical) 4-6</p> <p>8: $\forall x.R(x)$ \forall intro 3-7</p> <p>9: \perp \neg elim 8,1</p> <p>10: $\exists y.\neg R(y)$ contra (classical) 2-9</p>	<p>1: $\neg\exists x.R(x)$ premise</p> <p>2: $\neg\forall y.\neg R(y)$ assumption</p> <p>3: actual i assumption</p> <p>4: $R(i)$ assumption</p> <p>5: $\exists x.R(x)$ \exists intro 4,3</p> <p>6: \perp \neg elim 5,1</p> <p>7: $\neg R(i)$ \neg intro 4-6</p> <p>8: $\forall y.\neg R(y)$ \forall intro 3-7</p> <p>9: \perp \neg elim 8,2</p> <p>10: $\forall y.\neg R(y)$ contra (classical) 2-9</p>