Computer Science 81, Fall 2006
Assignment 10
Due Thur. Nov. 30

Proofs Involving Predicates

In the van Dalen book, please read pages from 100 to 108, 2nd paragraph. We're going to stop there and return to computability, using the Kozen book again.

It might also be a good idea to read the attached notes that clarify the practical usage of the quantifier rules. I have also included some notes on the Completeness Theorem.

For 1-7, first check for validity using the tree method. If valid, then prove using natural deduction. Otherwise, construct a counterexample. For the proofs, give both a tree and a box presentation.

1. $\forall x(\varphi(x) \to \psi(x)),\ \exists x(\sigma(x) \wedge \varphi(x)) \vdash \exists x(\sigma(x) \wedge \psi(x))$

2. $\varphi(a), \forall x(\varphi(x) \to \psi(x)) \vdash \psi(a)$

3. $\varphi(a), \exists x(\varphi(x) \to \psi(x)) \vdash \psi(a)$

4. $x = a \vee x = b \vdash \varphi(a) \vee \varphi(b)$

5. $\exists x \exists y\ (p(x, y) \vee p(y, x)) \vdash \exists x \exists y\ p(x, y)$

6. $\forall x\ p(x, x)$,
   $\forall x \forall y\ (p(x, y) \to p(y, x))$
   $\vdash \forall x \forall y \forall z\ ((p(x, y) \wedge p(y, z)) \to p(x, z))$

7. If anyone drinks coffee, then every mathematician does.
   If there are any programmers, then every coffee drinker is a programmer.

   Thus if there is a coffee-drinking programmer, then every mathematician is a programmer.

8. van Dalen, Page 99, Exercise 6.

9. van Dalen, Page 102, Exercise 4.