Solve the following problems in conjunction with reading Kozen lectures 6-12 (pages 33-76):

1. Kozen mentions that NFA’s with ε-transitions do not provide any additional power over NFA’s without ε-transitions. Re-formalize the definition of Δ-hat in Lecture 6 to accommodate ε-transitions. Make any necessary modifications to the remainder of the proof that for any NFA with ε-transitions there is a DFA accepting the same language.

2. Show that the reversal of a regular language is regular using structural induction on the definition of regular expressions.

3. For any language L, lastHalves(L) = {y | ∃x xy ∈ L and |x| = |y|}. Show that if L is regular, then so is lastHalves(L).

4. For any language L, define square(L) = {xx | x ∈ L}. Is square(L) regular for every regular language L? Prove your answer.

5. For any language L, define sqrt(L) = {x | xx ∈ L}. Is sqrt(L) regular for every regular language L? Justify your answer.