CS 141: Advanced Topics in Algorithms
Spring 2006
Homework Pyry and Topper

Due: Tuesday, April 18

1. [$\mathcal{R}_{OBL}(ALG)$ **Points**] **Stock Market Crash Part II!**

   The stock market problem is as follows: The stock market will crash sometime within a span of 100 days, with the limitation that it cannot crash on the first day, $t = 1$. There is only one stock, and its price rises linearly, so that $P(t) = t$, and the company ownes exactly one share. Each day the algorithm is told whether the market has crashed: if the market has crashed, and the algorithm has not sold, it makes zero profit, and the game ends. If the market has not crashed, the algorithm has the option of selling the one share for a profit of $P(t) = t$, at which point the game ends.

   We can use Yao's principle to determine a lower bound on the competitive ratio of any algorithm:

   $$\mathcal{R}_{OBL}(ALG) \geq \max\left\{\min_i \frac{\mathbf{E}_{y(j)}[OPT(\sigma_j)]}{\mathbf{E}_{y(j)}[ALG_i(\sigma_j)]}, \min_i \frac{1}{\mathbf{E}_{y(j)}\left[\frac{ALG_i(\sigma_j)}{OPT(\sigma_j)}\right]}\right\}$$

   In class we found half of the solution by showing that

   $$\min_i \frac{1}{\mathbf{E}_{y(j)}\left[\frac{ALG_i(\sigma_j)}{OPT(\sigma_j)}\right]} \geq 2.6$$

   with $y(j)$ being a unform distribution (that is, that the market is equally likely to crash on any day except the first).

   Complete the proof by evaluating

   $$\min_i \frac{\mathbf{E}_{y(j)}[OPT(\sigma_j)]}{\mathbf{E}_{y(j)}[ALG_i(\sigma_j)]}$$

   once again assuming that $y(j)$ is the uniform distribution over all possible crash days.