# CS157: Computer Animation

Motion controlling curves

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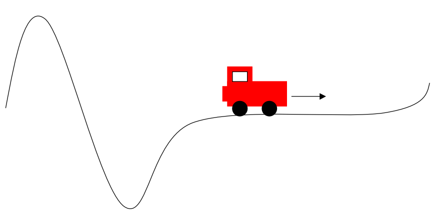
# Animation

Change over time

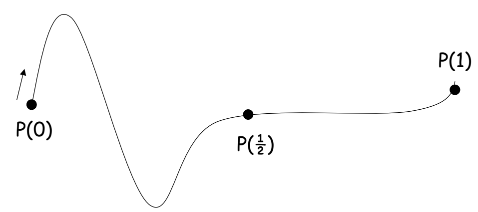
- Position
- Orientation
- Color
- Texture
- Shape
- Transparency
- Etc.

We can describe the change by a curve.

# Example: Motion along a curve

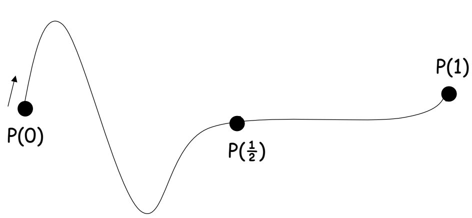


# Motion Along a Parameterized Curve



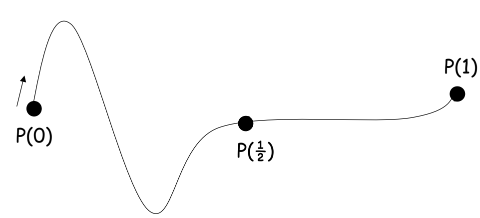

$P(t)=(X(t),Y(t))$ for $t \in [0,1]$

# Motion Along a Parameterized Curve




Question: What is the position of a particle at time $t$ if it starts at point P(0) at time $t=0$ and moves at constant speed $v$ along the curve?

# Re-parameterization

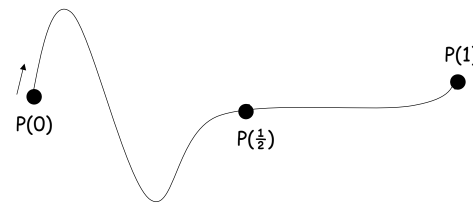



Find the point a distance D along the curve from P(0) in the direction of P(1).

# Re-parameterization

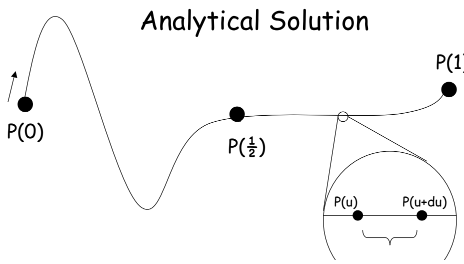
- Analytical method
- Numerical methods
- Forward Differencing

## Analytical Solution




Find u such that $\int_{0..u}$ ___________du = D

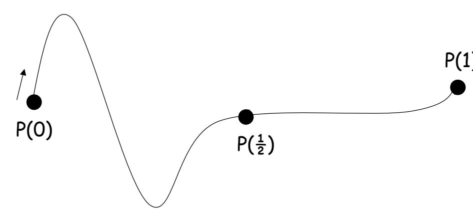
## Analytical Solution




the distance along the curve from P(u) to P(u+du) is approximately

$\sqrt{(X(u+du)-X(u))^2+(Y(u+du)-Y(u))^2}$

## Analytical Solution




Find u such that

$\int_{0..u}\sqrt{(dX(u)/du)^2+(dY(u)/du)^2}\,du = D$

# Analytical Solution

There is no general closed form solution!

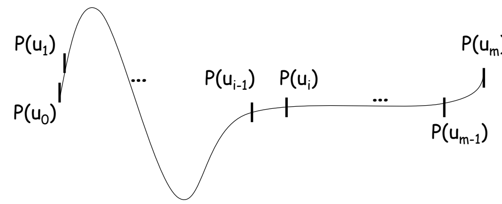
# Re-parameterization

- Analytical method
- **Numerical methods: Integration**
- Forward Differencing

# Re-parameterization

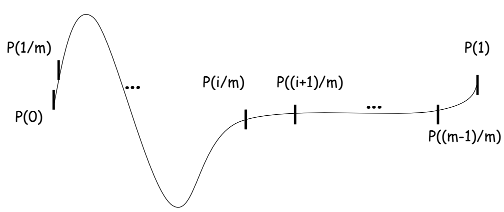
- Analytical method
- Numerical methods
- **Forward Differencing**

# Forward Differencing



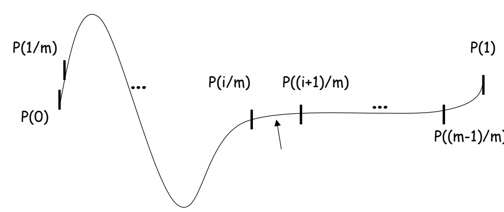

0. Choose a *good* step length *m*.

# Forward Differencing



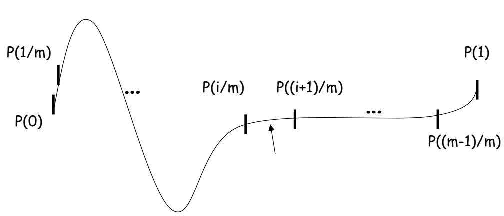

1. Sample P(u) at u=i/m for i=0,…,m

# Forward Differencing




2. Approximate curve length from P(i/m) to P((i+1)/m) as the distance $d_{i+1}$ between the points

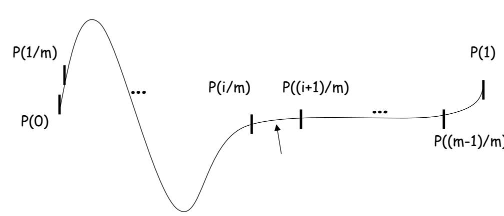
# Forward Differencing




for example, in 2D:

$$d_{i+1}=\sqrt{(X((i+1)/m)-X(i/m))^2 + (Y((i+1)/m)-Y(i/m))^2}$$

# Forward Differencing




3. Approximate curve length $D_i$ from P(0) to P(i/m) as the $\Sigma_{k=1..i} d_i$

## Re-Parameterization Table

| i/m | $d_i$ | $D_i$ |
|---|---|---|
| 0 | 0 | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| ⋮ | ⋮ | |
| 1.0 | .02 | 1.0 |

## Re-Parameterization Table

| i/m | $d_i$ | $D_i$ |
|---|---|---|
| 0 | 0 | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| ⋮ | ⋮ | ⋮ |
| 1.0 | .02 | 1.0 |

What is the distance from P(0) to P(u)?

## Re-Parameterization Table

| i/m | $d_i$ | $D_i$ |
|---|---|---|
| 0 | 0 | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| ⋮ | ⋮ | ⋮ |
| 1.0 | .02 | 1.0 |

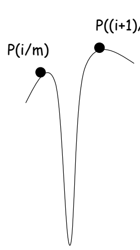
What is the distance from P(v) to P(u)?

## Re-Parameterization Table

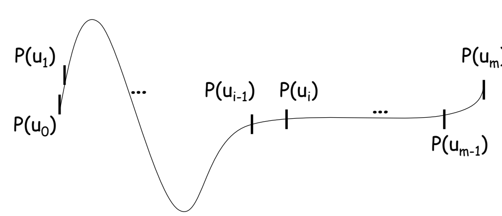
| i/m | $d_i$ | $D_i$ |
|---|---|---|
| 0 | 0 | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| ⋮ | ⋮ | ⋮ |
| 1.0 | .02 | 1.0 |

What point is a distance D along the curve from P(0)?

## Problem




## Forward Differencing

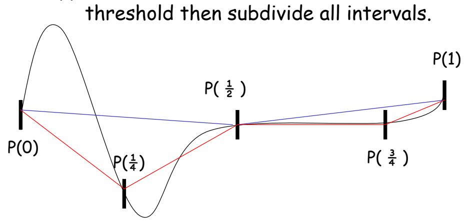



0. Choose a *good* step length *m*.

What is good choice for m?

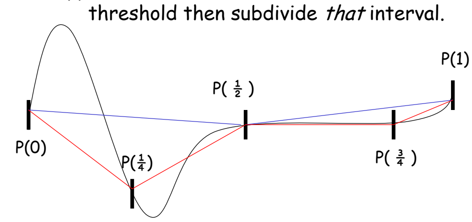
# Choosing m

If approximate error of an interval exceeds some threshold then subdivide all intervals.

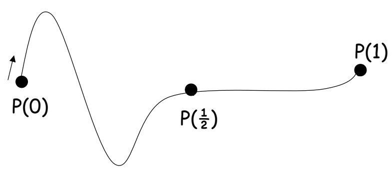



# Adaptive Forward Differencing

If approximate error of an interval exceeds some threshold then subdivide *that* interval.




## Motion Along a Parameterized Curve

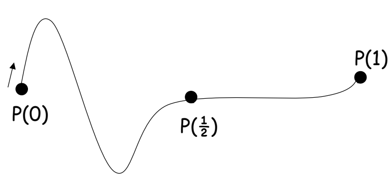



Question: What is the position of a particle at time *t* if it starts at point P(0) at time *t* =0 and moves at constant speed *v* along the curve?

Find parameter u such that P(u) is a distance *vt* from P(0).

| u | $d_i$ | $D_i$ |
|---|---|---|
| 0 | - | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| | | |
| 1.0 | .02 | D |

## Normalized Motion Along a Parameterized Curve

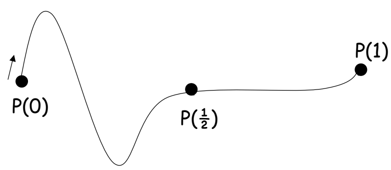



Question: What is the position of a particle at time $t \in [0,1]$ if it starts at point P(0) at time *t* =0, moves at constant speed, and reaches P(1) at time t=1?

Find parameter u such that P(u) is a distance *tD* from P(0).

| u | $d_i$ | $D_i$ |
|---|---|---|
| 0 | - | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| | | |
| 1.0 | .02 | D |

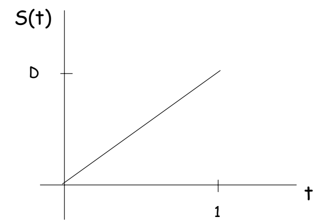
## Motion Along a Parameterized Curve




Question: What is the position of a particle at time *t* if it starts at point P(0) at time *t* =0, moves according to *speed control curve* S(t), and arrives at P(1) at time *t* =1?

## Speed Control Curve

at time t we've moved a distance S(t)

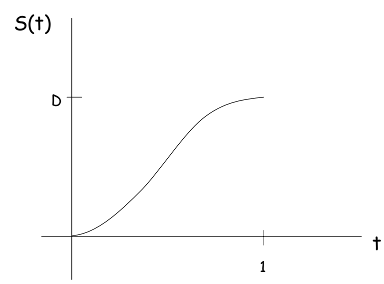



Find parameter u such that P(u) is a distance *S(t)* from P(0).

| u | $d_i$ | $D_i$ |
|---|---|---|
| 0 | - | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| | | |
| 1.0 | .02 | D |

## Ease in/Ease out

at time t we've moved a distance S(t)




Find parameter u such that P(u) is a distance *S(t)* from P(0).

| u | $d_i$ | $D_i$ |
|---|---|---|
| 0 | - | 0 |
| .05 | .12 | .12 |
| .10 | .09 | .21 |
| | | |
| 1.0 | .02 | D |