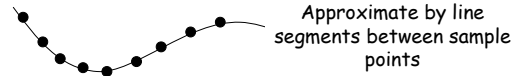
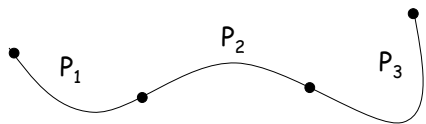


## CS157: Curves II

## Drawing Curves



## Complicated Curves



Simple curves connected end-to-end

## Simple Curves

How should we represent a simple curve?



- Flexibility
- Efficiency
- Usability
- Boundary constraints: Can we specify continuity (including derivatives) at boundaries?

## Parametric Continuity

$C^i$ : The 0<sup>th</sup>, 1<sup>st</sup>, 2<sup>nd</sup>, ...,  $i$ <sup>th</sup> derivative of adjacent curves agree at their boundary points.

## Geometric Continuity

$$G^0 = C^0$$

For  $i > 0$ ,  $G^i$  means

- $G^0$  continuity plus
- The 1<sup>st</sup>, 2<sup>nd</sup>, ...,  $i$ <sup>th</sup> derivative of adjacent curves are proportional at boundary point.

## Curve Representation

- Explicit
- Implicit
- **Parametric**

Cubic polynomials are good!

## Cubic Polynomial Parametric Curves

- interpolating curves
- hermitian splines
- catmull-rom
- bezier
- b-splines

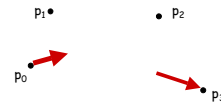
## Cubic Bezier Curves

- Input: Control points  $p_0, p_1, p_2, p_3$



## Cubic Bezier Curves

- Input: Control points  $p_0, p_1, p_2, p_3$
- Curve starts at  $p_0$  and ends at  $p_3$ .



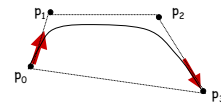
## Cubic Bezier Curves

- Input: Control points  $p_0, p_1, p_2, p_3$
- Curve starts at  $p_0$  and ends at  $p_3$ .
- Line segments  $p_1-p_0$  and  $p_3-p_2$  are tangent to the curve at, respectively,  $p_0$  and  $p_3$ .



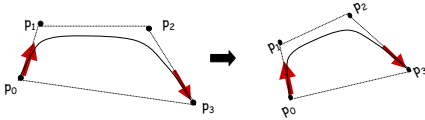
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- The curve lies within the convex hull of the control points.
- Curve is invariant under affine transformations.



## Parametric Bezier Curve

$$X(t) = at^3 + bt^2 + ct + d$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

## Exercise

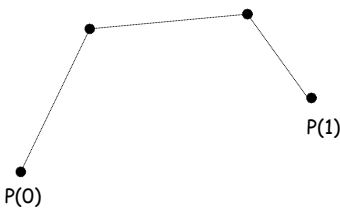
Find the blending functions for the bezier curve

## Drawing the Bezier curve

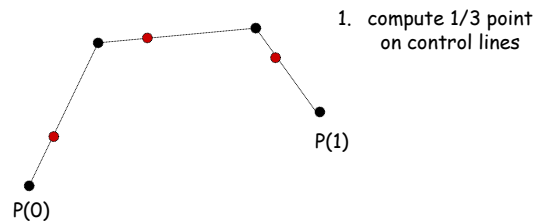
- Sample  $P(t) = (X(t), Y(t), Z(t))$  for  $t = t_0, \dots, t_n$
- Draw line segments from  $t_i$  to  $t_{i+1}$

What should n be?

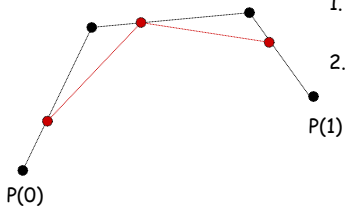
Drawing Bezier Curves  
deCasteljau's algorithm:  
compute  $P(1/3)$



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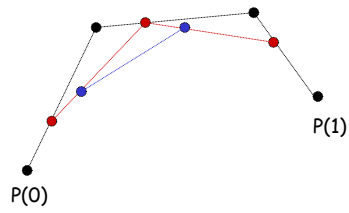


deCasteljau's algorithm:  
compute  $P(1/3)$



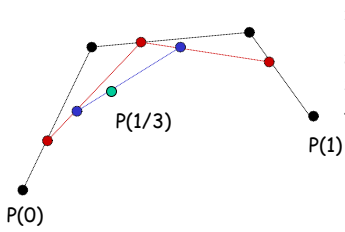
1. compute 1/3 point on control lines
2. connect

deCasteljau's algorithm:  
compute  $P(1/3)$



1. compute 1/3 point on control lines
2. connect
3. repeat

deCasteljau's algorithm:  
compute  $P(1/3)$



1. compute 1/3 point on control lines
2. connect
3. repeat
4. repeat

later

B-splines