A neural network for finding shortest paths in real time

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CS152 — Fall 2007

“Neural network based optimal routing algorithm for communication networks”

Venkataram, Ghosal, Kumar

Indian Institute of Science

2002

Problem formulation

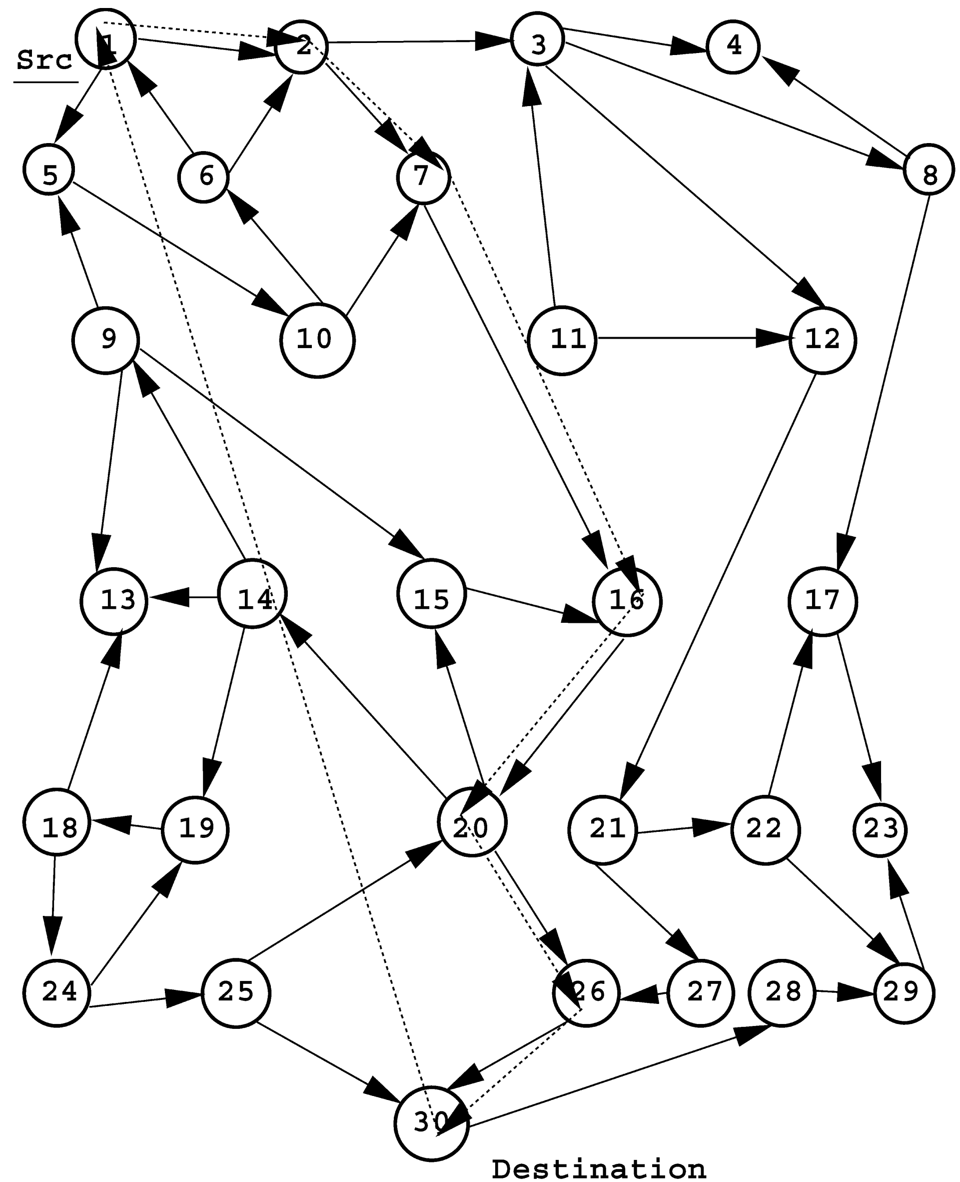
Input:

- $C_{xi} = \text{cost of arc } x \rightarrow i$
- $B_x = \text{cost of node } x$
- $\gamma_{xi} = \begin{cases} 1 & \text{arc } x \rightarrow i \text{ does not exist} \\ 0 & \text{otherwise} \end{cases}$
- source node s and destination d

Problem formulation

Output: $$V_{xi} = \begin{cases} 1 & \text{arc } x \rightarrow i \text{ is on output cycle} \\ 0 & \text{otherwise} \end{cases}$$

V will indicate a *cycle* consisting of the shortest path from s to d and the arc from d to s.



Src
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
Destination

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi}$$

edge used?

cost of edge

$$\frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi}$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

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$$\frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi}$$

Minimize total cost of arcs used

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi}$$

edge used?

edge used?

edge does not exist

$$\frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}\gamma_{xi}V_{xi}$$

$$E=\frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}C_{xi}V_{xi}+\frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}\gamma_{xi}V_{xi}+\frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}B_xV_{xi}+\frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n}V_{xi}-\sum_{i=1,i\neq x}^{n}V_{ix}\right\}^2+\frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n}V_{xi}(1-V_{xi})+\frac{\alpha_6}{2}(1-V_{ds})$$

edge used?

edge does not exist

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}\gamma_{xi}V_{xi}$$

Ensure we only use edges that exist

edge used?

$$\frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi}$$

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edge used?

cost of originating node

$$\frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi}$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n}\gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

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$$\frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi}$$

Minimize node cost

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} B_{xi}V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1, i\neq x}^{n} V_{xi} - \sum_{i=1, i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1 - V_{xi}) + \frac{\alpha_6}{2}(1 - V_{ds})$$

$$\frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1, i\neq x}^{n} V_{xi} - \sum_{i=1, i\neq x}^{n} V_{ix}\right\}^2$$

used arcs that leave node x

used arcs that enter node x

$$\frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_{xi}V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

zero iff the path exits every node it enters

used arcs that leave node x

used arcs that enter node x

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_{x}V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2$$

zero iff the path exits every node it enters

used arcs that leave node x

used arcs that enter node x

$$\frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2$$

Solution (V) indicates a cycle

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} B_{x}V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1, i\neq x}^{n} V_{xi} - \sum_{i=1, i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1-V_{xi})$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1, i\neq x}^{n} V_{xi} - \sum_{i=1, i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1-V_{xi})$$

$$V_{xi} \in \{0, 1\}$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_6}{2}(1-V_{ds})$$

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

$$\frac{\alpha_6}{2}(1-V_{ds})$$

Cycle includes arc from d to s

Minimize node cost

Minimize arc cost

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1,i\neq x,(x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1,i\neq x}^{n} V_{xi} - \sum_{i=1,i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1,x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

Ensure we only use edges that exist

Solution is a cycle that includes arc from d to s

Minimize $E \Rightarrow$

Minimize node cost

Minimize arc cost

$$E = \frac{\alpha_1}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} C_{xi}V_{xi} + \frac{\alpha_2}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} \gamma_{xi}V_{xi} + \frac{\alpha_3}{2}\sum_{x=1}^{n}\sum_{i=1, i\neq x, (x,i)\neq(d,s)}^{n} B_x V_{xi} + \frac{\alpha_4}{2}\sum_{x=1}^{n}\left\{\sum_{i=1, i\neq x}^{n} V_{xi} - \sum_{i=1, i\neq x}^{n} V_{ix}\right\}^2 + \frac{\alpha_5}{2}\sum_{i=1}^{n}\sum_{x=1, x\neq i}^{n} V_{xi}(1-V_{xi}) + \frac{\alpha_6}{2}(1-V_{ds})$$

Ensure we only use edges that exist

Solution is a cycle that includes arc from d to s

$\Rightarrow$ solve SP!

Parameter requirements

$$\alpha_1 < 2\frac{\alpha_4}{C_{max}}$$

$$\alpha_3 < 2\frac{\alpha_4}{B_{max}}$$

$$2\alpha_4 > \alpha_5$$

$$\alpha_2 = \alpha_6 \gg \alpha_1 C_{max} + \alpha_3 B_{max}$$

Network formulation

- Hopfield net
  - Each ordered pair of input vertices (possible directed edge) gets a network node

Network formulation

u_i input to node i

V_i output of node i

T_{ij} weight of connection $i \rightarrow j$

I_i bias of node i

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + \sum_{j=1}^{n} T_{ij} V_j + I_i$$

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + \sum_{j=1}^{n} T_{ij} V_j + I_i$$

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + \sum_{j=1}^{n} T_{ij} V_j + I_i$$

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} - \frac{\partial E}{\partial V_i}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} - \frac{\partial E}{\partial V_{xi}}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} - \frac{\partial E}{\partial V_{xi}}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} - \frac{\alpha_1}{2} C_{xi}(1 - \delta_{xd}\delta_{is}) - \frac{\alpha_2}{2}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} - \frac{\alpha_1}{2} C_{xi}(1-\delta_{xd}\delta_{is}) - \frac{\alpha_2}{2}\gamma_{xi}(1-\delta_{xd}\delta_{is}) - \frac{\alpha_3}{2} B_x(1-\delta_{xd}\delta is) - \alpha_4 \sum_{y=1, y\neq x}^{n} (V_{xy} - V_{yx}) + \alpha_4 \sum_{y=1, y\neq i}^{n} (V_{iy} - V_{yi}) - \frac{\alpha_5}{2}(1-2V_{xi}) + \frac{\alpha_6}{2}\delta_{xd}\delta_{is}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} - \frac{\alpha_1}{2} C_{xi}(1 - \delta_{xd}\delta_{is}) - \frac{\alpha_2}{2}\gamma_{xi}(1 - \delta_{xd}\delta_{is}) - \frac{\alpha_3}{2} B_x(1 - \delta_{xd}\delta is) - \alpha_4 \sum_{y=1, y\neq x}^{n} (V_{xy} - V_{yx}) + \alpha_4 \sum_{y=1, y\neq i}^{n} (V_{iy} - V_{yi}) - \frac{\alpha_5}{2}(1 - 2V_{xi}) + \frac{\alpha_6}{2}\delta_{xd}\delta_{is}$$

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau_{xi}} + \sum_{y=1}^{n} \sum_{j=1, j\neq y}^{n} T_{xi,yj} V_{yj} + I_{xi}$$

$$T_{xi,yj} = \alpha_5 \delta_{xy} \delta ij - \alpha_4 \delta_{xy} - \alpha_4 \delta_{ij} + \alpha_4 \delta jx + \alpha_4 \delta_{iy}$$

$$I_{xi} = \begin{cases} \frac{\alpha_6 - \alpha_5}{2} & \text{if } (x, i) = (d, s) \\ -\frac{\alpha_1}{2} C_{xi} - \frac{\alpha_2}{2} \gamma_{xi} - \frac{\alpha_3}{2} B_x - \frac{\alpha_5}{2} & \text{otherwise} \end{cases}$$

Simulations

Simulations

- Always “valid solutions” within 4-8k iterations

Simulations

- Always “valid solutions” within 4-8k iterations
- No real-time performance metrics…