

cs155 - z sweedyk

# 3D graphics scene graphs

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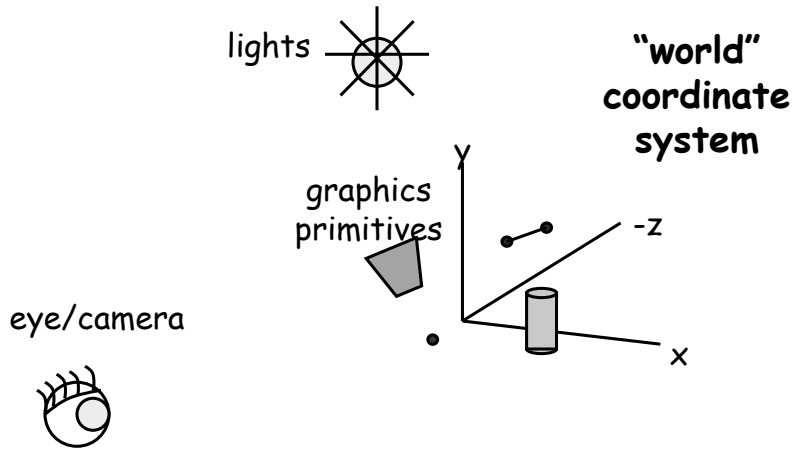
## overview

- brief overview of 3D
- modeling transforms & homogenous coordinates
- hierarchical coordinates

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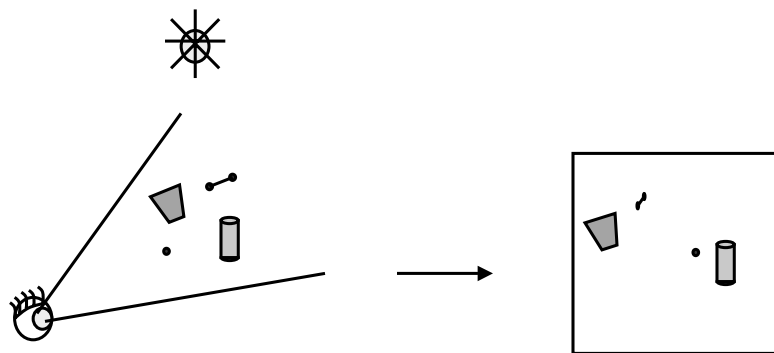
# 3d scene



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# rendering



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## overview

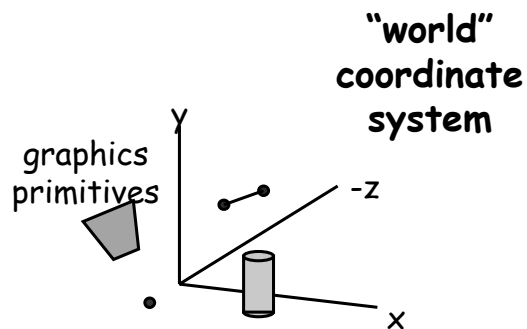
- brief overview of 3D
- modeling transforms & homogenous coordinates
- hierarchical coordinates

DONE

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## today: composing the scene



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## geometric primitives

- points
- lines
- triangles
- spheres

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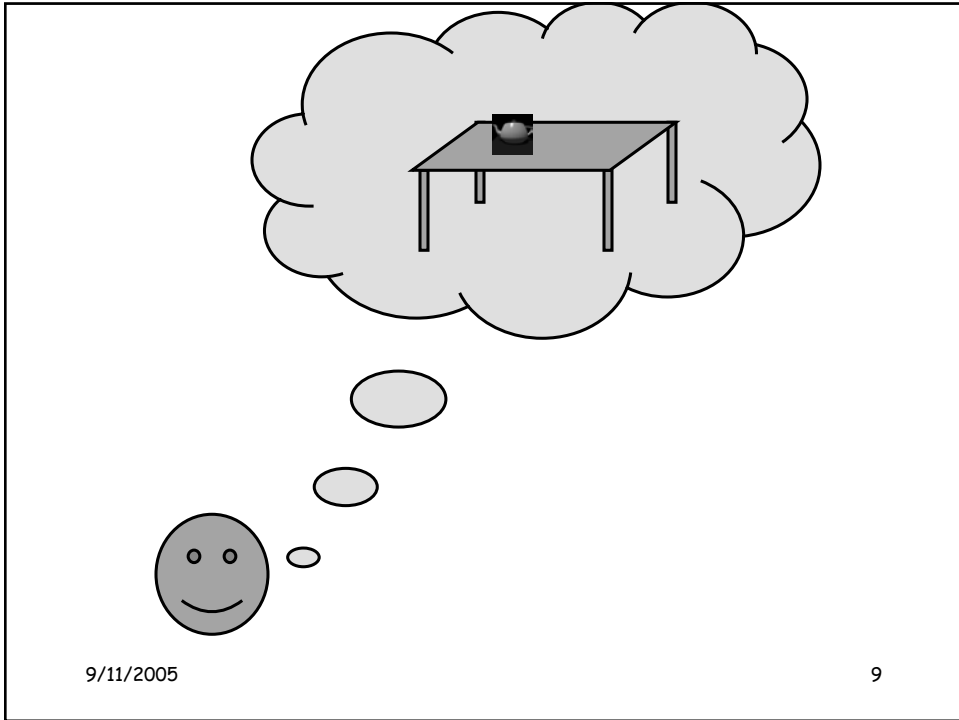
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## triangle mesh



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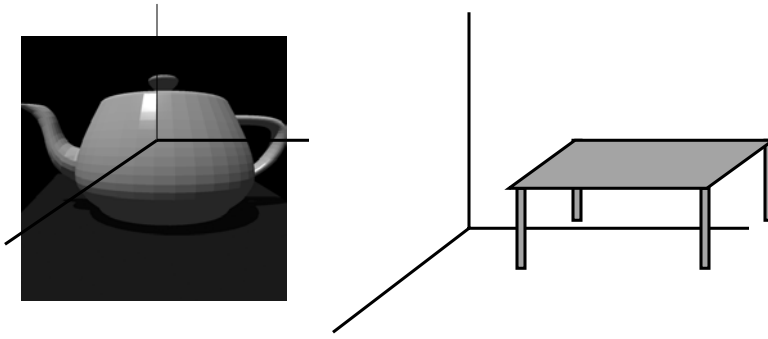
## 3d model store

receipt
one triangle mesh teapot
one triangle mesh table

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# putting it together

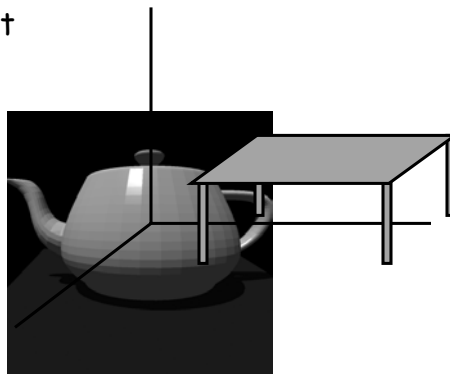


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# putting it together

call Martha Stewart



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## making it fit

transform teapot

scale by .1

translate by (20,10,2)

## making it fit

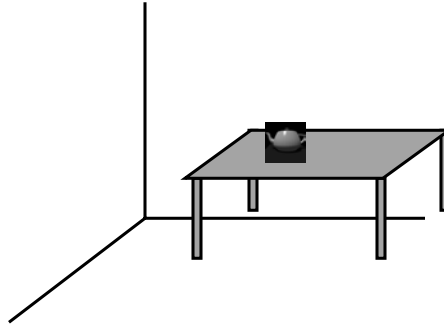
for each triangle in the teapot

scale by .1

translate by (20,10,2)

will this work?

scale and translate teapot triangles ...



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now rotate it a little

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# transforms

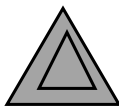
- scale
- rotate
- translate

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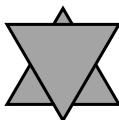
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# triangle

scale



rotate



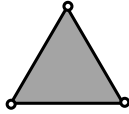
translate



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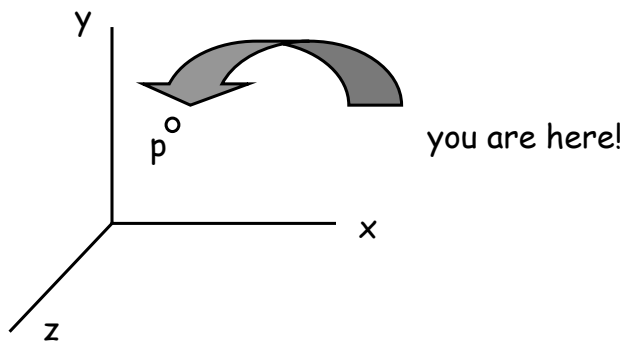
triangle



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points



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# point

call prof gu

scale



rotate



translate



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# linear algebra

- scalars
- vectors

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## scalars: real numbers

3.8

2.7

4.1

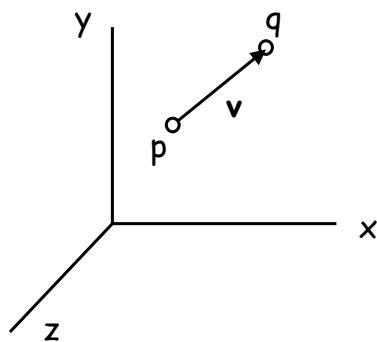
-1000.2

5

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## vector: magnitude & direction in (3d) space

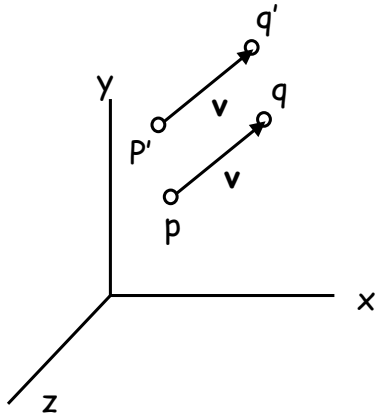


$v$ : the way you get from  $p$  to  $q$

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vector: magnitude & direction in (3d) space

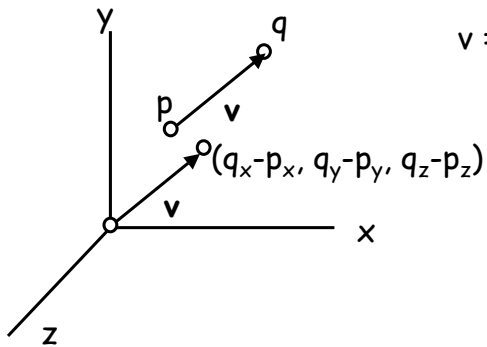


A vector does not have a position in space !

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naming vectors



$$v = \langle q_x - p_x, q_y - p_y, q_z - p_z \rangle$$

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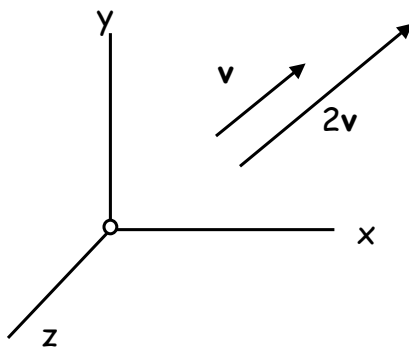
## linear spaces

- scalars
- vectors
- scalar multiplication
- vector addition

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## scalar multiplication

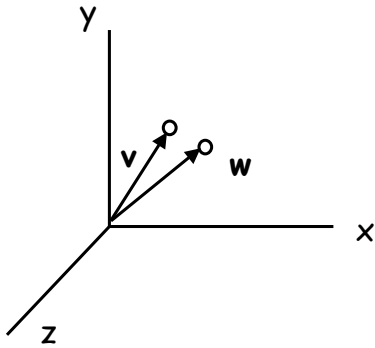


$$2v = \langle 2v_x, 2v_y, 2v_z \rangle$$

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## vector addition



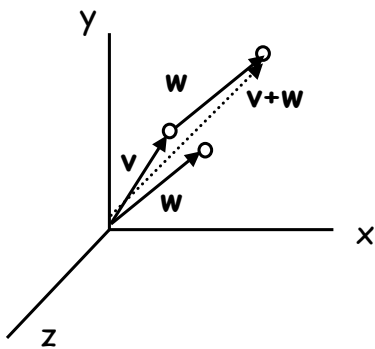
$$v = \langle v_x, v_y, v_z \rangle$$

$$w = \langle w_x, w_y, w_z \rangle$$

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## vector addition



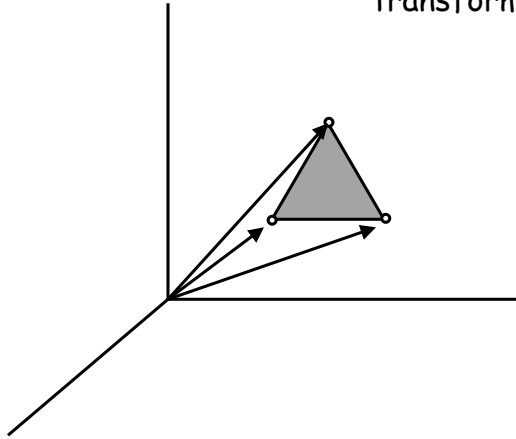
$$v+w = \langle v_x+w_x, v_y+w_y, v_z+w_z \rangle$$

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triangle

transform vectors!

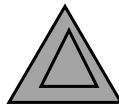


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triangle

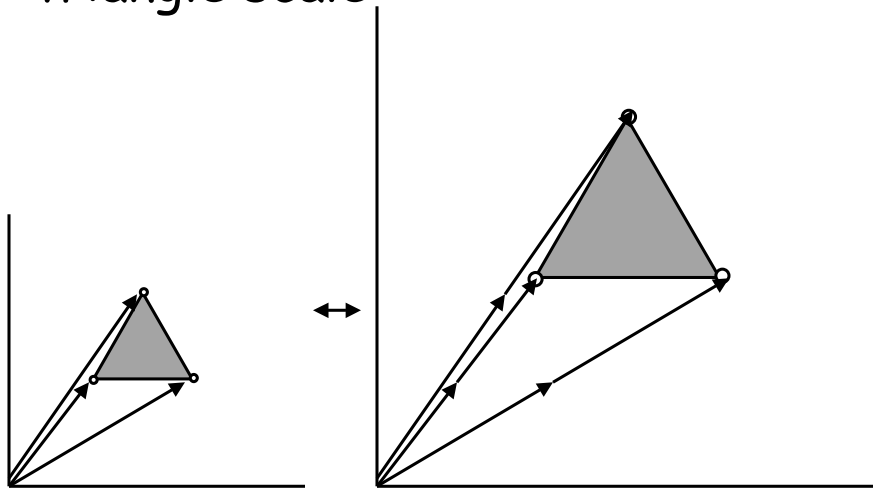
scale



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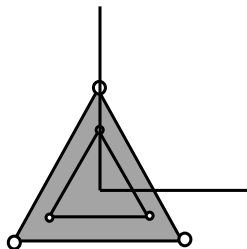
# triangle scale



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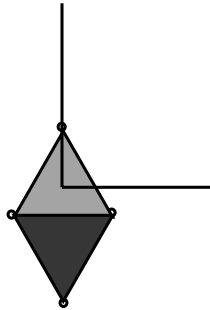
# triangle scale



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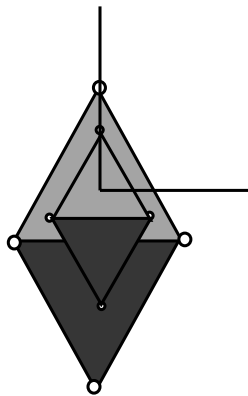
# triangle scale



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# triangle scale



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## scale

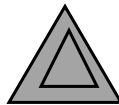
$$\begin{pmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & u \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} sx \\ ty \\ uz \end{pmatrix}$$

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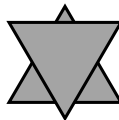
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## triangle

scale



rotate

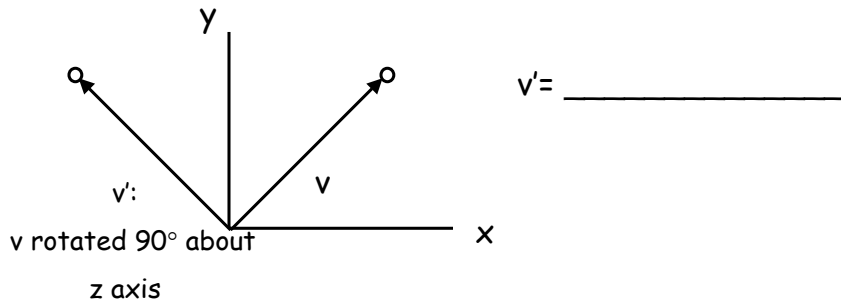


translate

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## rotating a vector



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## rotation

$$\begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} v_x \cos 90 - v_y \sin 90 \\ v_x \sin 90 + v_y \cos 90 \\ v_z \end{bmatrix}$$

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## rotate about z axis

$$\begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- the first column specifies what happens to (1,0,0)

- the second column specifies what happens to (0,1,0)

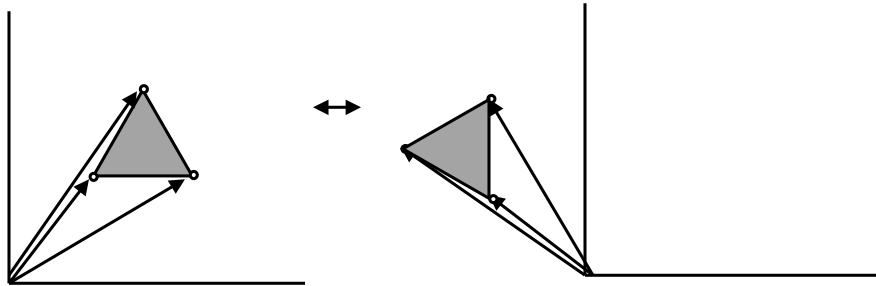
- the third column specifies what happens to (0,0,1)

## rotation in 3D about x axis

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

fill in the blanks

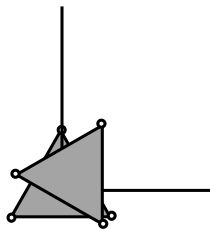
# triangle rotate



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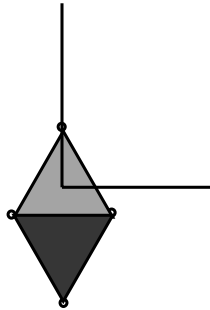
# triangle rotate



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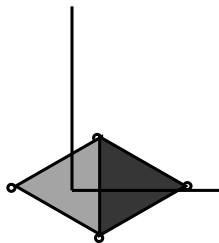
triangle rotate



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triangle rotate



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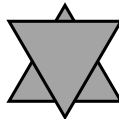
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# triangle

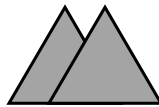
scale



rotate



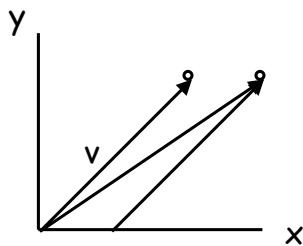
translate



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# translating a point



$p$  translated 2 units  
to the right

$$f(\langle x, y, z \rangle) = \langle x+2, y, z \rangle$$

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## operators

- scale
  - rotate
- } linear
- translate
- non-linear

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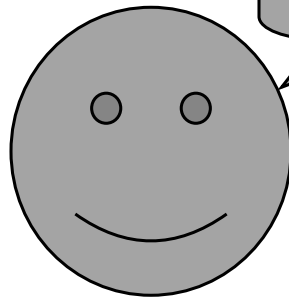
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## linear transformation

- $f(\mathbf{v})$  is linear if
$$f(\alpha\mathbf{u}+\beta\mathbf{v}) = \alpha f(\mathbf{u})+\beta f(\mathbf{v})$$
- translation is not a linear transform:  
define  $T(\mathbf{v}) = \mathbf{v}+\mathbf{w}_0$  where  $\mathbf{w}_0$  is a non-zero vector
$$T(\mathbf{u}+\mathbf{v}) = \mathbf{u} + \mathbf{v} + \mathbf{w}_0$$
$$T(\mathbf{u})+T(\mathbf{v}) = \mathbf{u}+\mathbf{v}+2\mathbf{w}_0$$

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But dude ... we know  
how to translate  
points!

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## composite transform

$$M_1 v \longrightarrow M_2(M_1 v)$$

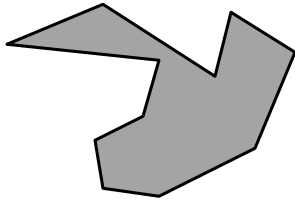


$$(M_2 M_1) v$$

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# transform polygon mesh



1,000,000  
vertices  
10 transforms

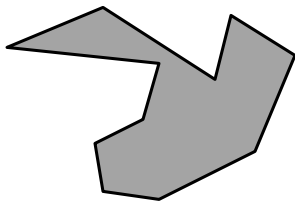


10,000,000  
computations

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# transform polygon mesh



1,000,000  
vertices  
~~10 transforms~~  
1 composite transform



1,000,000  
~~10,000,000~~  
computations

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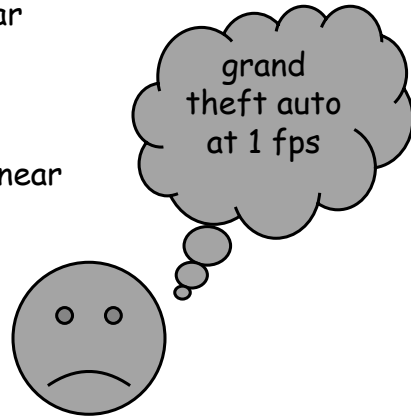
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## operators

- scale
  - rotate
- }
- translate

linear

non-linear



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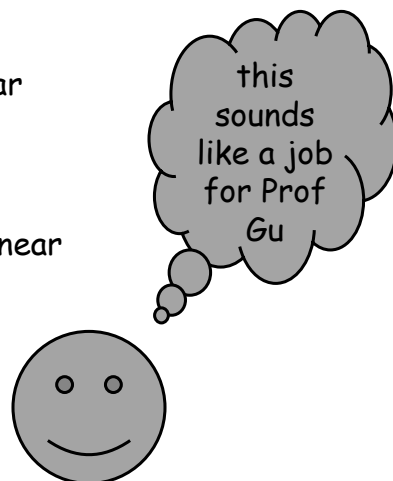
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## operators

- scale
  - rotate
- }
- translate

linear

non-linear



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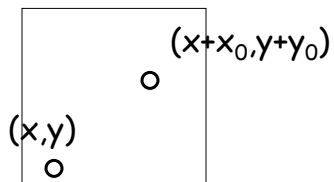
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let's step back into 2D for a moment

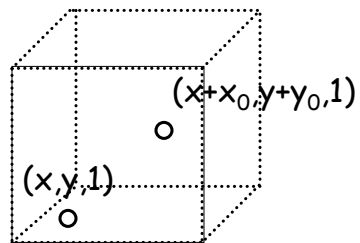
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hold onto your hat ...



not a 2D linear  
xfm



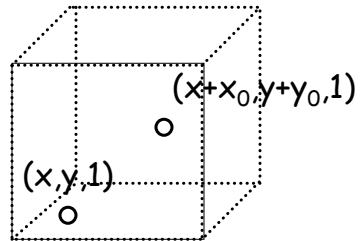
3D linear xfm

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## linear transform

$$\begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+x_0 \\ y+y_0 \\ 1 \end{pmatrix}$$



NOTE: this only works as a translate for points in the plane  $z=1$ !

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## 2D homogenous coordinates

$$(x, y) \iff (x, y, 1)$$

Can we do scale and rotate in homogenous coordinates?

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## scale

$$\begin{pmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} sx \\ ty \\ 1 \end{pmatrix}$$

## rotate

$$\begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \cos \phi - y \sin \phi \\ x \sin \phi + y \cos \phi \\ 1 \end{pmatrix}$$

translate

$$\begin{pmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+x_0 \\ y+y_0 \\ 1 \end{pmatrix}$$

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transform form

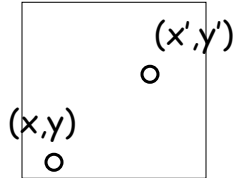
$$\begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ \hline 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

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# we are not alone...

the parallel universe view of homogenous coordinates



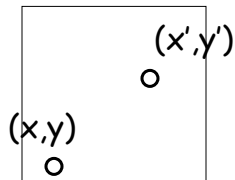
we live in this universe

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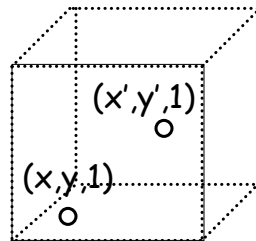
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# we are not alone...

the parallel universe view of homogenous coordinates



we live in this universe



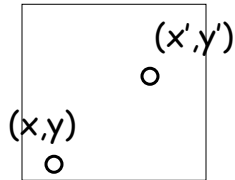
it's not the only one, but  
it is the only one we can  
experience!

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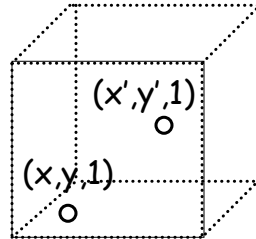
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and its better not to think about it ...

the parallel universe view of homogenous coordinates



our universe has  
center (0,0)

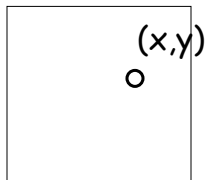


center?

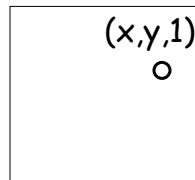
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## 2d and 2d homogenous



our universe

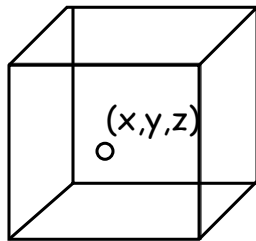


our universe when  
it comes to  
computing modeling  
transforms

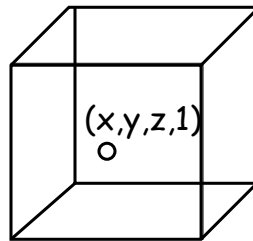
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## 3d and 3d homogenous



our universe



our universe when  
it comes to  
computing modeling  
transforms

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## scale

$$\begin{pmatrix} s & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} sx \\ ty \\ uz \\ 1 \end{pmatrix}$$

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## rotate about z axis

$$\begin{pmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \cos \phi - y \sin \phi \\ x \sin \phi + y \cos \phi \\ z \\ 1 \end{pmatrix}$$

rotate about x & y axes are similar

## translate

$$\begin{pmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x+x_0 \\ y+y_0 \\ z+z_0 \\ 1 \end{pmatrix}$$

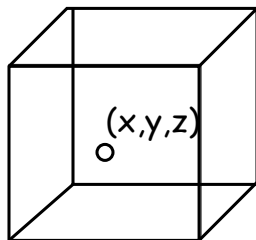
## transform form

$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix}$$

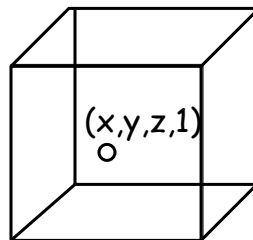
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## 3d and 3d homogenous



our universe

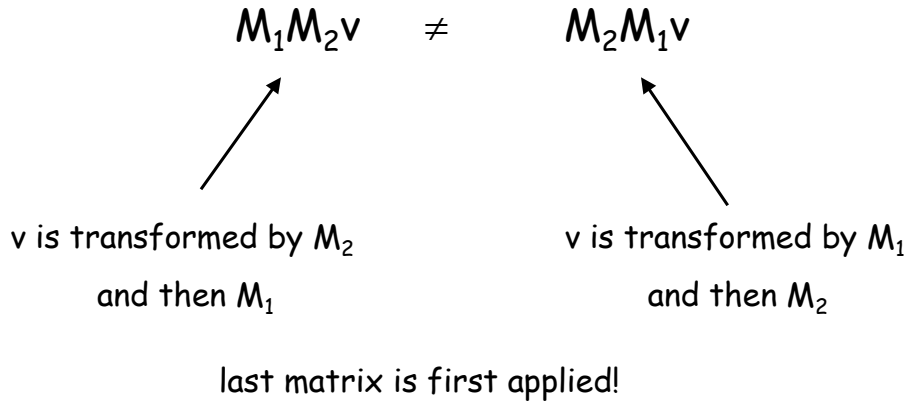


our universe when  
it comes to  
computing modeling  
transforms

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## Composing transforms



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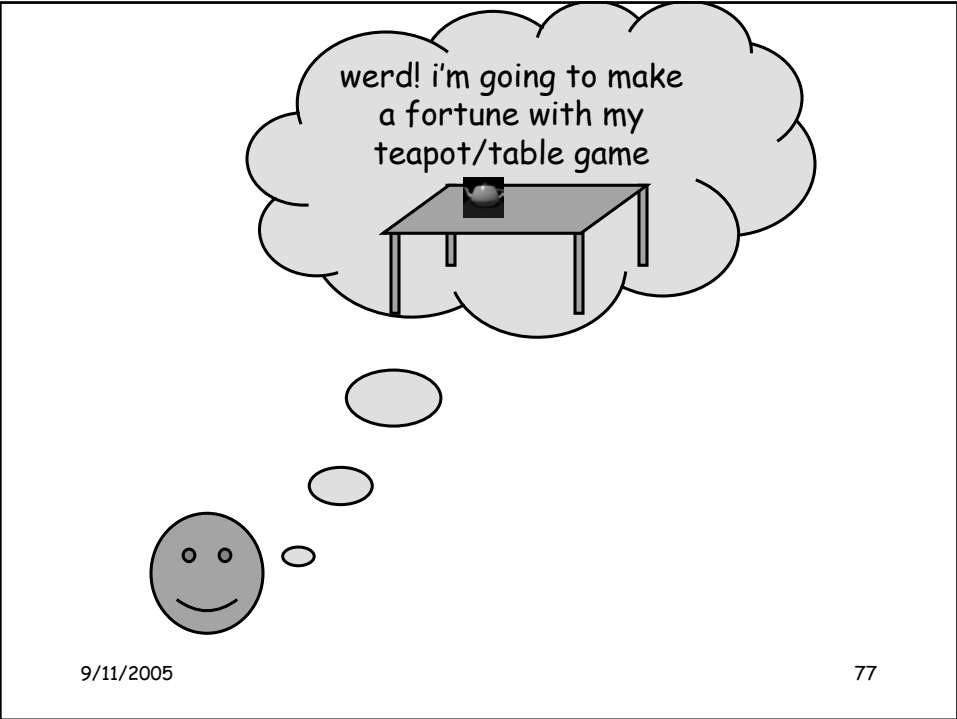
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## overview

- brief overview of 3D
- modeling transforms & homogenous coordinates DONE
- hierarchical coordinates

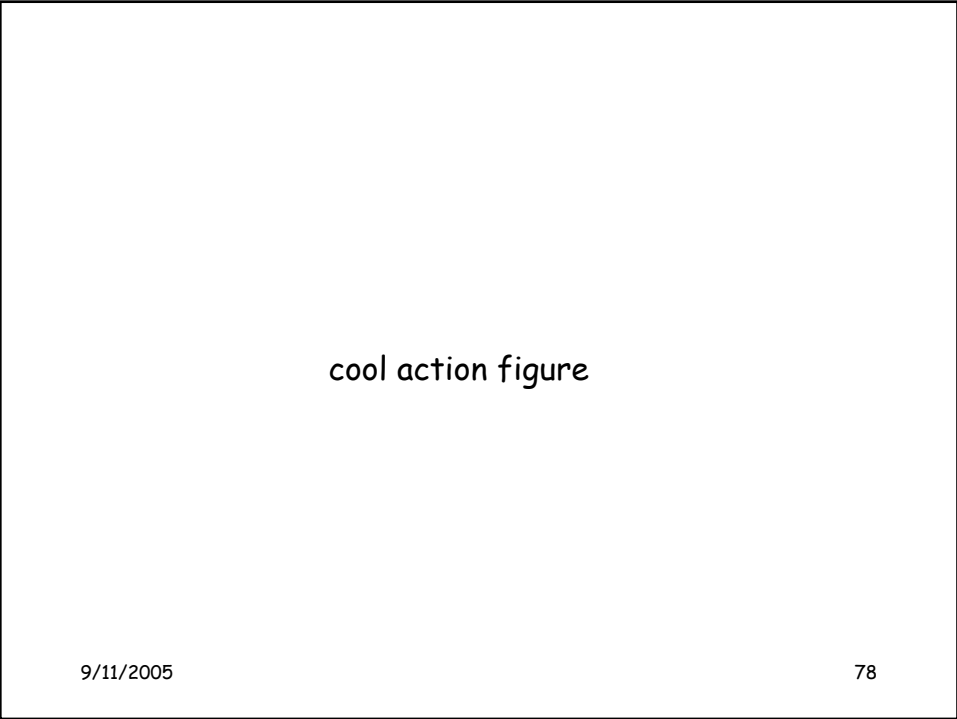
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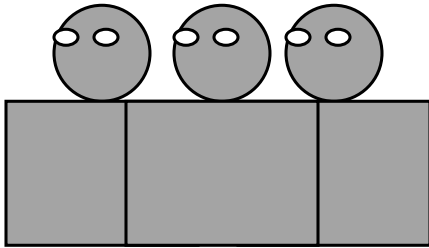


cool action figure

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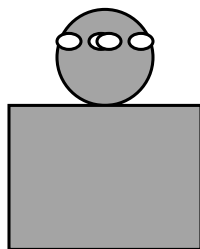
cool action figure



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cool action figure



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## 3D model store

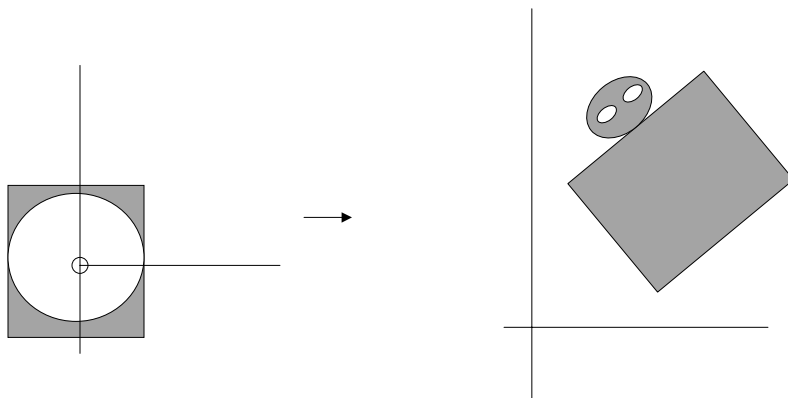
receipt	
one orange cube	← body
one orange sphere	← head
two white sphere	← eyes

cube has side length one and is centered at the origin  
spheres have radius one and are centered at the origin

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## transformations

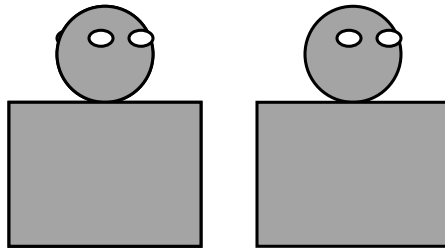


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# problem

transform components as independently and as one  
(in real time)

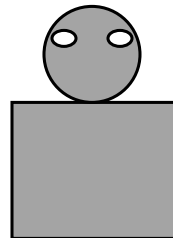


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# scene graph

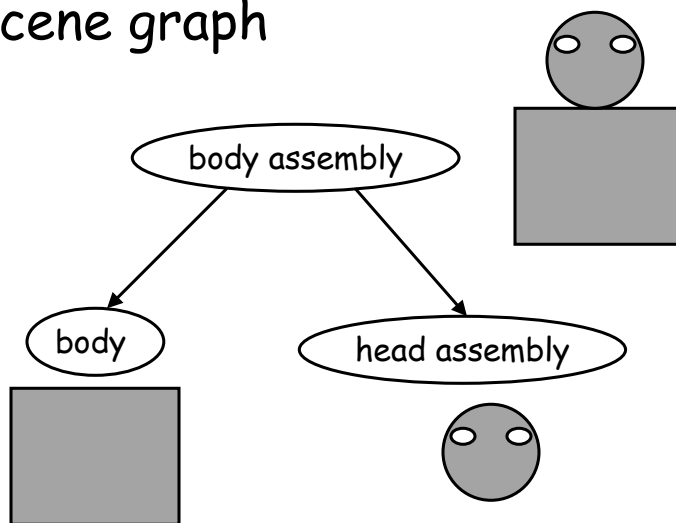
body assembly



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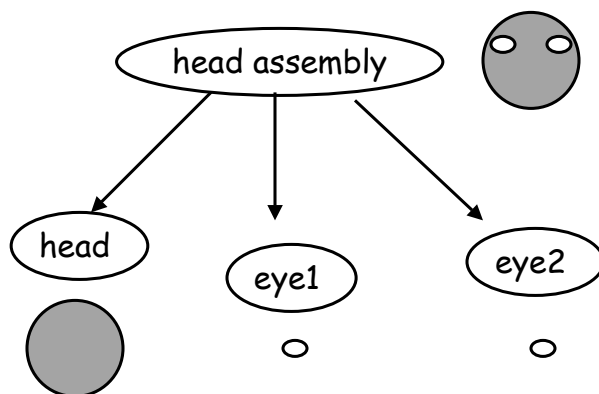
# scene graph



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# scene graph

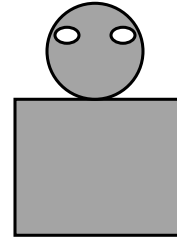
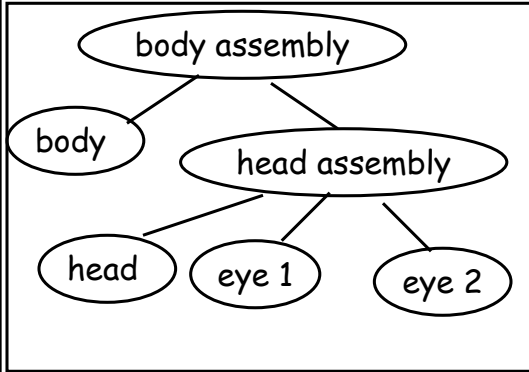


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# scene graph

transformation

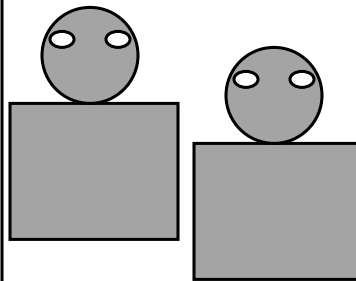
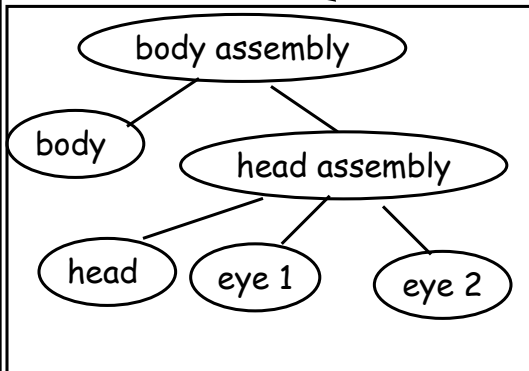


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# scene graph

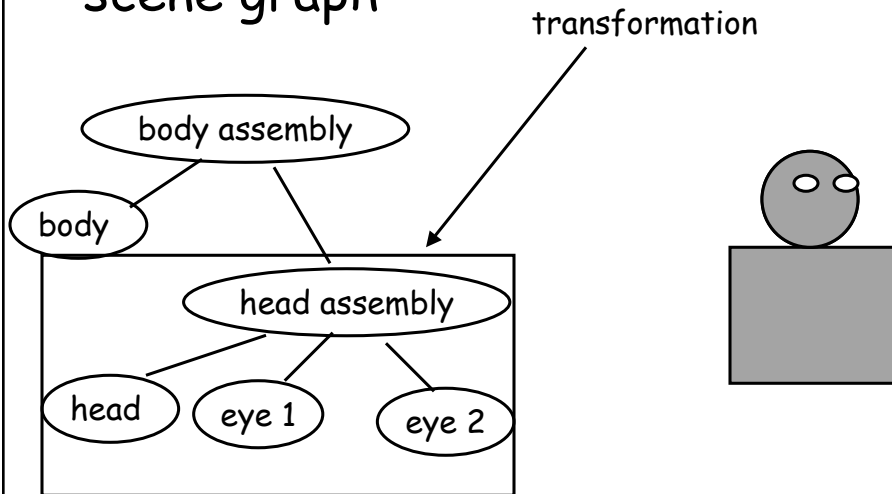
transformation



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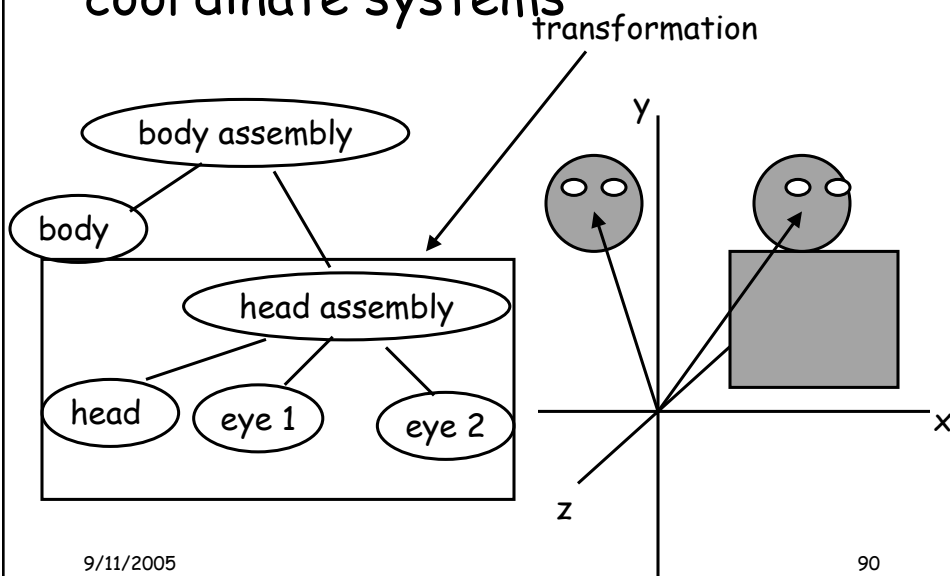
# scene graph



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# coordinate systems

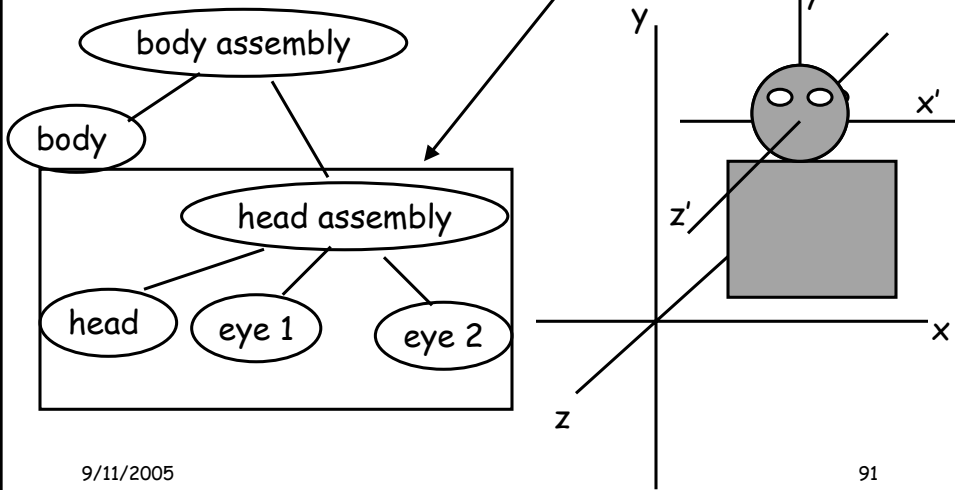


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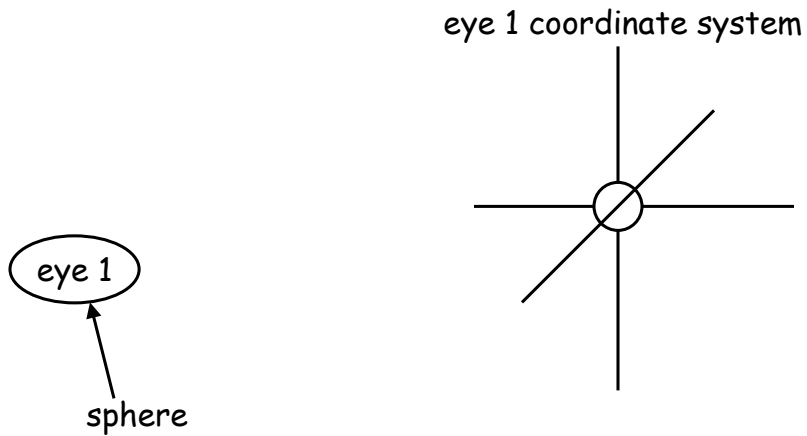
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# coordinate systems

transformation in LOCAL coordinate system

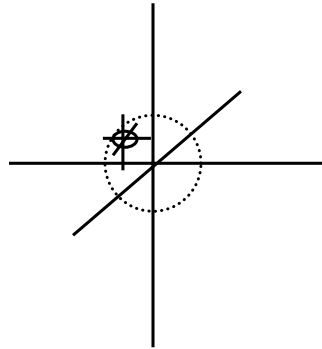
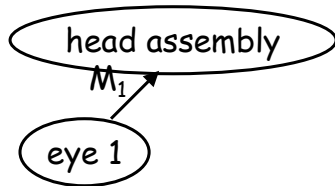


# hierarchical coordinates



# hierarchical coordinates

head assembly coordinate system



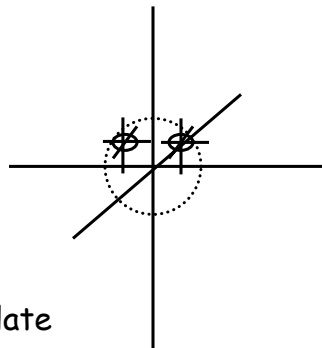
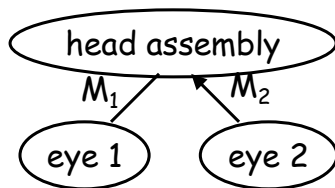
$M_1$ : scale and translate

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# hierarchical coordinates

head assembly coordinate system



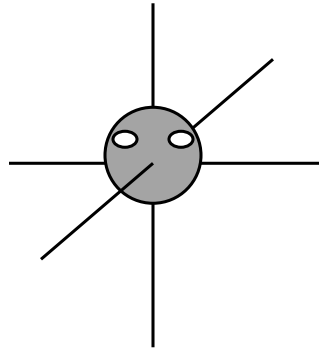
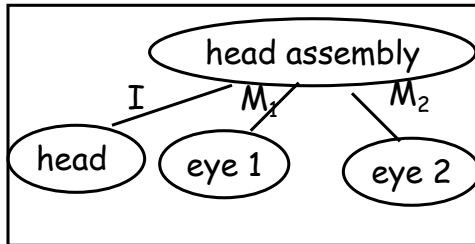
$M_2$ : scale and translate

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# hierarchical coordinates

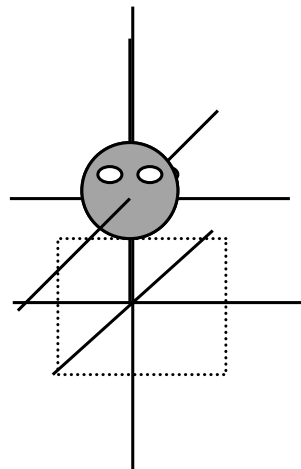
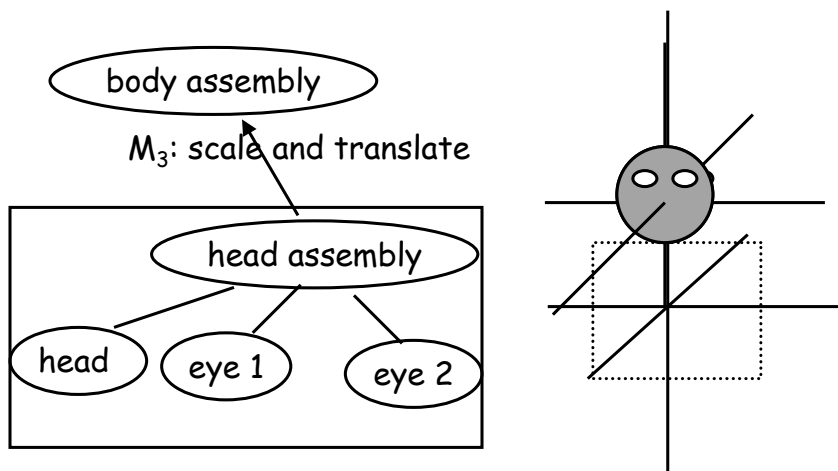
head assembly coordinate system



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# hierarchical coordinates

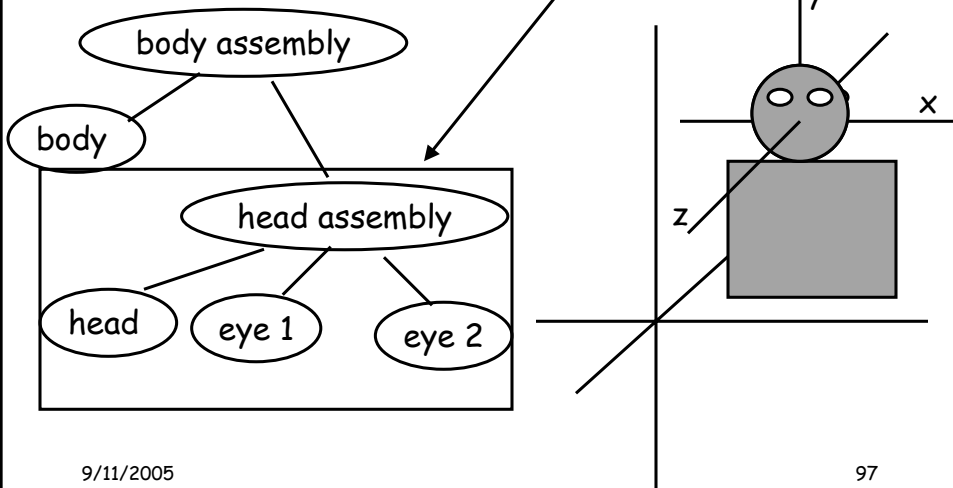


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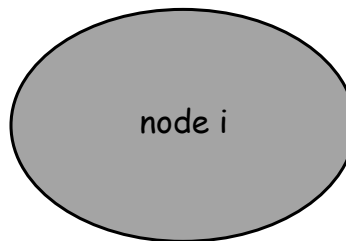
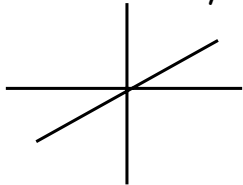
# hierarchical coordinates

rotate in your LOCAL coordinate system



# scene graph node

node i coordinate system

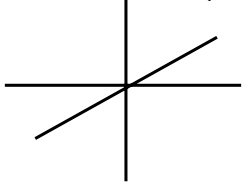


geometric primitives

defined in node i's  
coordinate system

# scene graph node

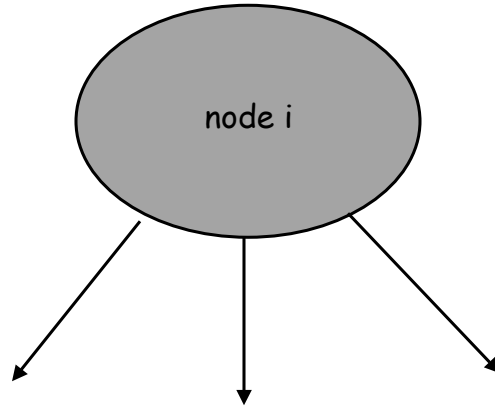
node i coordinate system



geometric primitives  
defined in node i's  
coordinate system

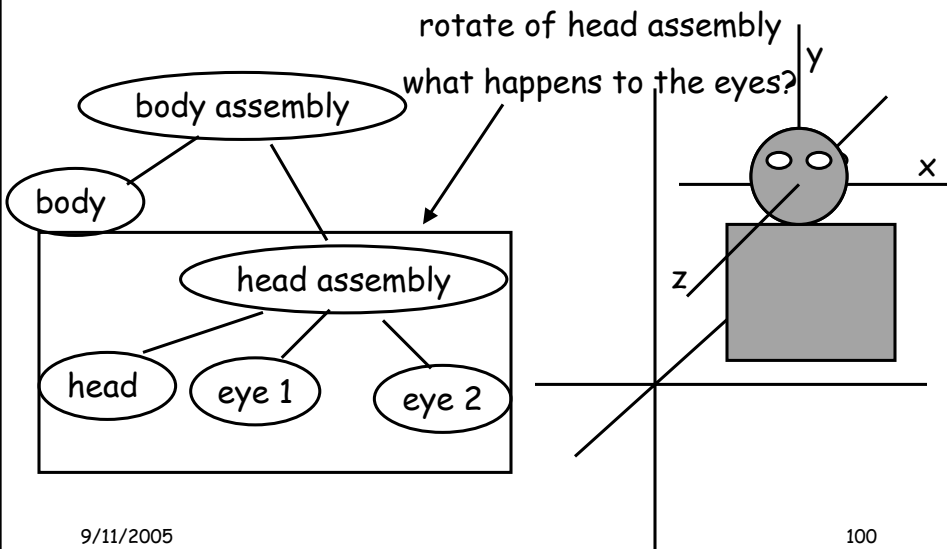
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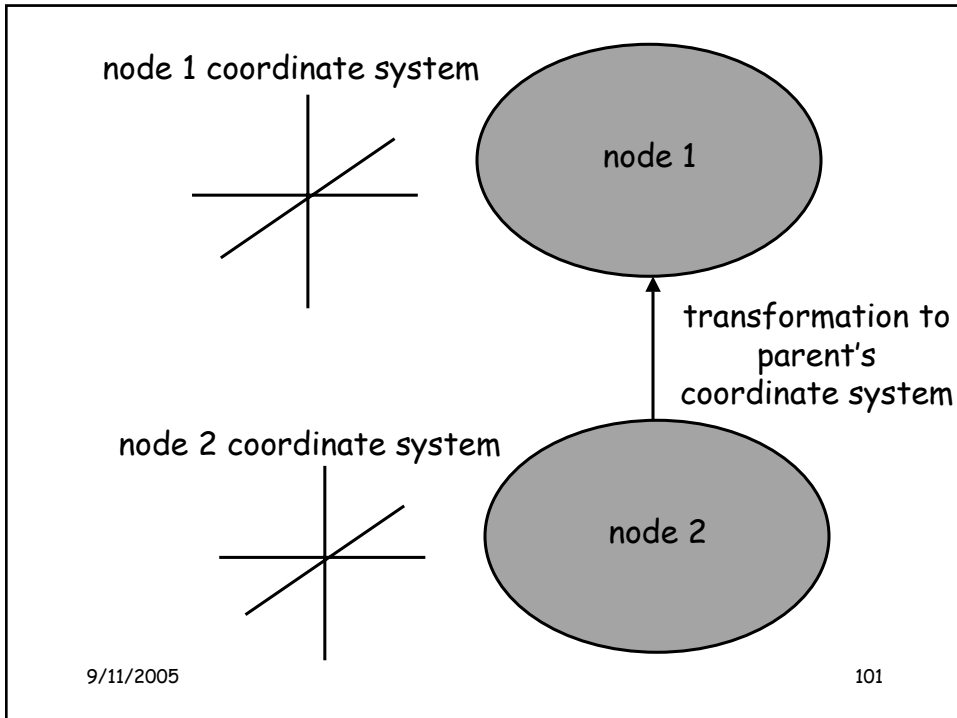
transform node i and its subtree  
relative to node i's coordinate system



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# hierarchical coordinates





## hierarchical coordinates

// Left eye

- Transform to head assembly coordinate system:
  - `translate(-1,1,0)`  $M_1$
  - `scale(0.2,0.2,0.2)`  $M_2$
- Sphere (radius 1, centered at origin)

## hierarchical coordinates

// Right eye

- Transform to head assembly coordinate system:
  - translate(1,1,0)  $M_4$
  - scale(0.2,0.2,0.2)  $M_3$
- Sphere (radius 1, centered at origin)

## hierarchical coordinates

// Head

- Sphere (radius 1, centered at origin)

## hierarchical coordinates

- // Head assembly
  - Transform to body assembly coordinate system
    - Translate(0,1/2,0)  $M_7$
    - Scale(.4,.4,.4)  $M_6$
  - Local transform
    - Rotate( $\theta$ ,y-axis)  $M_5$
  - Children
    - Left eye
    - Right eye
    - Head

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## hierarchical coordinates

- // Body
  - Cube

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# hierarchical coordinates

// Body assembly

• Local transform

• Translate(dx,dy,dz)  $M_8$

• Children

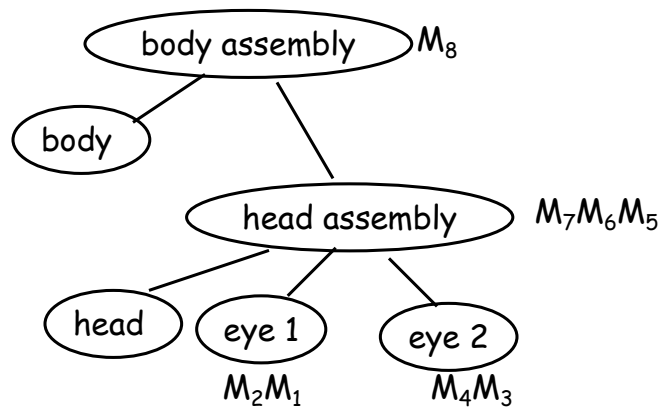
• Body

• Head assembly

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# hierarchical coordinates



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## Exercise

What is the composite transform applied to left eye?

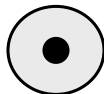
What is the composite transform applied to right eye?

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## Exercise

Suppose we used a texture mapped sphere  
for the eye:



How would we modify the scene graph so  
that we could roll the eyes?

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Exercise: build the scene graph for a snowman that has a head, middle, bottom, two eyes, nose, and mouth.

## snowman: what you can use

- spheres: radius 1, centered at origin  
(color options: white, black, red)
- scale (s,t,u)
- rotate (theta, vector)
- translate (dx,dy,dz)

download rt & snowman.ray  
expand snowman.ray to include your scene graph!  
render with rt