CS 81 Logic and Computability

Fall 2007

HW 11 due 12/13/07

1. Rice's theorem states that no non-monotonic property of r.e. sets is semi-decidable.
    a. Give three examples of non-monotonic properties of r.e. sets.
    b. Let P be any non-monotonic property of r.e. sets and let $L_1 \subseteq L_2$ be r.e. sets such that L_1 has property P and L_2 does not. Let M be some TM and let w be any string. Show how to construct a new Turing machine M′ such that:

        If M halts on w then $L(M') = L_2$

        Otherwise $L(M')=L_1$
    c. Show that recognizing L_{NHP}={M,w | M does not halt on w} reduces to the problem of recognizing L_P={M | L(M) has property P}.

2. Determine whether the following sets are (a) recursive, (b) recursively enumerable but not recursive, or (c) not recursively enumerable. Prove your answers. (Do not use Rice's theorem.)
    a. {M | L(M) is context free}
    b. {M | $L(M)=L(M)^R$}
    c. {M | M takes more than 20 steps on all input}
    d. {M,i | M enter state q_i on some input}

3. Prove that a language L is recursively enumerable iff there is a generator G that enumerates the strings in L.

4. Prove that a language L is recursive iff there is a generator G that enumerates the strings in L in canonical order.

5. Let L_1 and L_2 be recursive languages. Classify the following problems as decidable, semi-decidable, or undecidable. Prove your answers.

    a. Is $L_1 \neq L_2$?
    b. Is x in L_1-L_2?
    c. Is $L_1=L_2$?