CS 81 Logic and Computability

Fall 2007

HW 7 due 11/13/07

1. Prob. 10, p. 683 Hein

2. Prove the following *improved* pumping lemma. Let L be any regular language. Then there exists an n such that for any x in L, for any way of writing x as $z_1z_2z_3$ with $|z_2|\geq n$, there exists strings w_1, w_2, and y such that $w_1yw_2=z_2$, $|w_1y|\leq n$, $|y|>0$, and the string $z_1w_1y^kw_2z_3$ is in L for all $k\geq 0$.

3. Which of the following languages over A={a,b} are regular? Prove your answer. (Hint: You may find the improved pumping lemma particularly helpful for at least one of these problems.)

   a. $\{a^{3n} \mid n\geq 1\}$
   b. $\{a^ib^ja^k \mid i>j>k\}$
   c. $\{a^ib^ja^k \mid i+j+k \text{ is even}\}$
   d. $\{xwx^R \mid x\neq\varepsilon\}$ where x^R denotes the string x in reverse order
   e. $\{x \mid x=x^R\}$
   f. $\{xx^Rw \mid x\neq\varepsilon\}$

4. Let L be a regular language. Which of the following are also regular? Prove your answer.
   a. $L^R=\{x \mid x^R \varepsilon L\}$
   b. $½ L = \{x \mid \exists y\ |y|=|x| \text{ and } xy\ \varepsilon\ L\}$
   c. $\{x \mid xx^R\ \varepsilon\ L\}$
   d. $\{xz \mid \exists y\ |y|=|x|=|z| \text{ and } xyz\ \varepsilon\ L\}$

5. Prove the following theorem.

   Let M be a finite automaton with n states. Then L(M) is
   i. nonempty if and only M accepts a string of length less than n
   ii. infinite if and only if M accepts a string of length m where $n\leq m\leq 2n$.

EXTRA CREDIT:

Let L be a regular language. Prove that $Sqrt(L) = \{x \mid \exists y\ |y|=|x|^2 \text{ and } xy\ \varepsilon\ L\}$ is regular.