1. Prob. 10, p. 683 Hein

2. Prove the following improved pumping lemma. Let \( L \) be any regular language. Then there exists an \( n \) such that for any \( x \) in \( L \), for any way of writing \( x \) as \( z_1z_2z_3 \) with \( |z_2| \geq n \), there exists strings \( w_1 \), \( w_2 \), and \( y \) such that \( w_1yw_2 = z_2 \), \( |w_1y| \leq n \), \( |y| > 0 \), and the string \( z_1w_1y^kw_2z_3 \) is in \( L \) for all \( k \geq 0 \).

3. Which of the following languages over \( A=\{a,b\} \) are regular? Prove your answer. (Hint: You may find the improved pumping lemma particularly helpful for at least one of these problems.)
   a. \( \{a^{3n}\mid n \geq 1\} \)
   b. \( \{a^ib^ka^k \mid i > j > k\} \)
   c. \( \{a^ib^ka^k \mid i+j+k \text{ is even}\} \)
   d. \( \{xwx^R \mid x \neq \epsilon\} \) where \( x^R \) denotes the string \( x \) in reverse order
   e. \( \{x \mid x=x^R\} \)
   f. \( \{xx^Rw \mid x \neq \epsilon\} \)

4. Let \( L \) be a regular language. Which of the following are also regular? Prove your answer.
   a. \( L^R=\{x \mid x^R \in L\} \)
   b. \( \frac{1}{2} L = \{x \mid \exists y \mid |y|=|x| \text{ and } xy \in L\} \)
   c. \( \{x \mid xx^R \in L\} \)
   d. \( \{xz \mid \exists y \mid |y|=|x|=|z| \text{ and } xyz \in L\} \)

5. Prove the following theorem.
   Let \( M \) be a finite automaton with \( n \) states. Then \( L(M) \) is
   i. nonempty if and only \( M \) accepts a string of length less than \( n \)
   ii. infinite if and only if \( M \) accepts a string of length \( m \) where \( n \leq m \leq 2n \).

EXTRA CREDIT:
Let \( L \) be a regular language. Prove that \( \text{Sqrt}(L) = \{x \mid \exists y \mid |y|=|x|^2 \text{ and } xy \in L\} \) is regular.