CS 81 Logic and Computability

Fall 2007

HW 9 due 11/27/07

1. (25 pts.) Consider the following grammar:
   $S \rightarrow aAbb \mid bBaa \mid A \mid T$
   $A \rightarrow aB$
   $B \rightarrow bA$
   $T \rightarrow bTaa \mid S \mid \varepsilon$
   a. Give an equivalent grammar with no useless symbols, no ε-productions, and no unit productions.
   b. Convert the resulting grammar to Chomsky normal form.
2. (25 pts.) A grammar is in Greibach normal form if all productions have the form $A \rightarrow aB_1B_2 \ldots B_k$ where a is a terminal symbol, $B_1B_2 \ldots B_k$ are variables, and $k \geq 0$.
   a. Give an algorithm for converting a grammar in Chomsky normal form to Greibach normal form.
   b. Apply the algorithm to the grammar you found in 1b.

3. (25 pts.) Which of the following languages are context-free? Prove your answer.
   a. $\{a^ib^jc^kd^m \mid i=j \text{ and } k=m\}$
   b. $\{a^ib^jc^kd^m \mid i=m \text{ and } j=k\}$
   c. $\{a^ib^jc^kd^m \mid i=k \text{ and } j=m\}$

4. (25 pts.) Construct a PDA that recognizes each of the following languages
   a. $\{xy \mid x,y \in \{a,b\}^* \text{ and } x \neq y\}$
   b. Strings with an equal number of a's and b's.