1. Construct a DFA that recognizes each of the following languages. Justify your answer by describing, in English, the purpose of each state.

   a. The strings over \{p,q,r,\neg,\land,\lor,(,)\} that represent well formed formula in conjunctive normal form (CNF) with no unnecessary parenthesis. In other words the strings have the form \(C_1\land C_2\land\ldots\land C_n\) where each \(C_i\) has the form \((x_1\lor x_2\lor\ldots x_k)\), with \(x_i\) in \{p,\neg p,q,\neg q,r,\neg r\}. The empty clause is acceptable as is a formula with an empty clause (\()).

   b. The strings over \{a,b\} in which the parities of a and b are the same; i.e. a string in the language has an even number of both a and b or an odd number of both a and b.

   c. The strings over \{a,b\} in which each consecutive block of 5 symbols has at least two b’s; i.e. if \(s_0s_1\ldots s_n\) is in the language then for any \(i, 0 \leq i \leq n-4\), the substring \(s_is_{i+1}s_{i+2}s_{i+3}s_{i+4}\) contains at least 2 b’s.

2. Construct an NFA that recognizes the following languages. Justify your answer by describing, in English, the purpose of each state.

   a. The strings over \{a,b\} containing at least 2 a’s and such that some pair of a’s is separated by a string whose length is a multiple of 3.

   b. The strings over \{a,b,c\} that have the same value when multiplied from the left as from the right according to the following (non-associative) multiplication table:

   \[
   \begin{array}{ccc}
   a & b & c \\
   a & a & a & c \\
   b & c & a & b \\
   c & b & c & a \\
   \end{array}
   \]

   For example, aaa is in the language because multiplication from the left yields \((aa)a=aa=a\) and from the right yields \(a(aa)=aa=a\). But bac is not because \((ba)c=cc=a\) but \(b(ac)=bc=b\).

3. Construct a regular expression for each of the following languages. Provide justification of correctness.

   a. The set of strings over \{0,1\} that represent, in binary, a number that is equivalent to zero modulo 3.

   b. The strings over \{a,b\} with an equal number of a’s and b’s such that in every prefix the number of a’s and the numbers of b’s differs by at most 2; i.e. if \(s_0s_1\ldots s_n\) is in the language then for any \(i, 0 \leq i \leq n\), the prefix \(s_0s_1\ldots s_i\) has the property that the number of a’s and b’s differ by at most 2.