Harvey Mudd College
Computer Science 81
Computability and Logic

Assignment 1
Languages and Regularity

Due Wed. 24 January 2007
at start of class

Relevant reading: Kozen, Lectures 2-9

Collaboration policy for this assignment: You may discuss problems with others. You may get help from the professor and tutors. However, the solution write-ups must be your own, in your own words. You may not look at or transcribe written material from others to enable writing your own solution. In other words, the solution you submit is based on your own assimilated knowledge. If there are any doubts as to interpretation of this policy, please ask before you act.

If you use hand-written submissions, write clearly. The graders may deduct for lack of readability.

[100 points total]

1. [5 points] Define three broad classes of computational problems.

2. [10 points] Define what is meant by the concept of a *language?* Discuss the relationship between language and the problem classes mentioned in 1.

3. [20 points] Which of the following statements are true for languages L, M, N? Justify each answer. ("Justify" means: If a statement is true, give a proof or argument for it. If the statement is false, give a specific counterexample. I give an example of a write-up on the next page.)

   a. If $L \subseteq M$ then $LN \subseteq MN$ (juxtaposition denotes concatenation)

   b. $L \subseteq L^*$ (* is the asterate or Kleene star operator)

   c. If $L \subseteq M$ then $L^* \subseteq M^*$.

   d. $L^*M^* \subseteq (LM)^*$.

   e. $L^*M^* \subseteq (L \cup M)^*$.

4. [5 points] What does it mean for a language to be *regular* (i.e. a *regular set*)?

5. [5 points] What is meant by the *complement* of a language? Is the complement of a regular language necessarily regular? Why, or why not.

6. [10 points] *Regular expressions* are a notation for defining regular languages. They are defined inductively using certain bases and certain operators. What are the bases, and what are the operators? Why is complement not included in the operators?

7. [15 points] The *symmetric difference* of two languages is defined by

$$L \oplus M = \{x \in L \mid x \notin M\} \cup \{x \in M \mid x \notin L\}.$$

Show that the symmetric difference of two regular languages is regular.

8. [15 points] Give a method for determining whether two regular languages, represented by regular expressions, are equal.

9. [15 points] Consider the language equation

$$L = ML \cup N$$

where L is regarded as a language variable, and M and N are given languages. Show that this equation has a solution $L = M^*N$.

Sample proof write-up for 3a:

If $L \subseteq M$ then $LN \subseteq MN$ (juxtaposition denotes concatenation)

Proof:

1. Assume $L \subseteq M$ (to show $LN \subseteq MN$).
2. Let x be an arbitrary element of LN.
3. By definition of concatenation, there are strings $u \in L$ and $v \in N$ such that $x = uv$.
4. By assumption 1, $u \in M$.
5. Thus $uv \in MN$, from 3 and 4.
6. Thus $x \in MN$, from 3 and 5.
7. Since an arbitrary element x of LN is also an element of MN, $LN \subseteq MN$.