Computer Science 81, Spring 2007
Assignment 11
Due Wed. April 25
Computability and Recursive Functions

1. [5 points] Suppose that e is the index of a 3-ary function. Show $S_{1n}(e, 0)$ as a composition, and show its index in the recursive formalism of rules R1-R7.

2. [15 points] The text indicates that the S_{mn} function can be derived by applying S_{1n} m times. Show in detail how this is possible.

3. [10 points] Show by diagonalization that the set of all total recursive functions (on natural numbers) of one argument is not recursively enumerable.

4. [10 points] Suppose that f is a unary recursive function from the natural numbers that is a one-to-one correspondence (for every $x \in N$ there is a unique $y \in N$ such that $f(x) = y$, and vice-versa). Show that the inverse function f^{-1} is recursive, where $f^{-1}(y)$ is the x such that $f(x) = y$.

5. [10 points] Suppose that R is a 2-ary recursively-enumerable relation on the natural numbers. Show that the set $\{y \mid \exists x\ R(x, y)\}$ is recursively-enumerable. (Think twice before using the μ operator.)

6. [10 points] Consider the language
   L = {<M> | M is a Turing machine that halts on at most 999 distinct inputs}.
   Is L recursively-enumerable? Justify your answer.

7. [10 points] Consider the language
   L = {<M> | M diverges on some input of length 999 or less}.
   Is L recursively-enumerable? Justify your answer.

8. [30 points] Using natural deduction, prove the following formulas in PA (Peano arithmetic) from the axioms listed on pages 236-237 of van Dalen:

   a. $\vdash x+y = y+x$
   b. $\vdash x(yz) = (xy)z$

   You will need to use the induction axiom, as listed. You may lump all rules for equality (not listed on pp 237) into a single rule, Eq.