Computer Science 81, Spring 2007

Assignment 9

Due Mon. April 9

Resolution

1. [10 points] Establish the satisfiability or unsatisfiability of the following sets of clauses by resolution. If a set is satisfiable, give a valuation that shows this.

   a. $\{p \vee q,\ \neg q \vee r,\ \neg r \vee \neg p\}$

   b. $\{p \vee q,\ p \vee \neg r,\ \neg p \vee r,\ \neg p \vee \neg q,\ \neg q \vee r,\ q \vee \neg r\}$

2. [10 points] Determine whether each pair of terms is unifiable, and if so, give the most general unifier:

   a. p(f(x, g(y)), y) *vs.* p(f(g(a), z), b)

   b. p(x, f(y, a), y) *vs.* p(f(a, b), v, z)

   c. p(x, f(x)) *vs.* p(g(a), f(h(a)))

3. [20 points] Establish the satisfiability or unsatisfiability of the following sets of formulas by resolution. If a set is satisfiable, give an interpretation that shows this.

   a. $\{p(x),\ q(y, a) \vee \neg p(y),\ \neg q(b, a)\}$

   b. $\{p(a),\ q(y, a) \vee \neg p(a),\ q(b, x)\}$

   c. $\{q(x),\ \neg p(y) \vee \neg p(g(a)) \vee \neg q(a),\ p(z) \vee \neg q(w)\}$

4. [20 points] Transform each formula into an equivalent set of clauses.

   a. $\exists y\ \forall x\ (p(x, y) \rightarrow q(x))$

   b. $\forall x\ \exists y\ (p(x, y) \vee \exists z\ q(x, y, z))$

5. [40 points] Prove each formula by the resolution method:

   a. $\forall x\ (p(x) \rightarrow q(x)) \rightarrow ((\forall x\ p(x)) \rightarrow (\forall x\ q(x)))$

   b. $\forall x\ (p(x) \rightarrow q(x)) \rightarrow ((\exists x\ p(x)) \rightarrow (\exists x\ q(x)))$

   c. $\exists x\ (p(x) \rightarrow \forall x\ p(x))$

   d. $((\exists x\ p(x)) \rightarrow (\forall x\ q(x))) \rightarrow (\forall x\ (p(x) \rightarrow q(x)))$

6. Extra credit [40 points]: Check each result 1, 2, 4, 5 using Otter-λ.