Your name _____

Harvey Mudd College
CS 81 Mid-Term Exam
Fall semester, 2006

Four sections on seven pages, 12 problems
100 Points
Closed book

Instructions

The exam is closed book. Do not refer to materials during the examination, other than a 2-sided 8.5 x 11" crib-sheet as described previously. Do not use a computer, calculator, or pda (personal digital assistant).

Please provide answers to the problems directly on these pages. Some problem pages are printed on the back to allow a 2-page facing space for your answer.

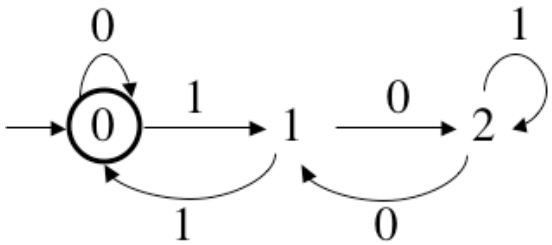
The exam has a time limit of 75 minutes. While you might not finish, work so as to maximize total points, avoiding getting stuck on one problem at the expense of others. It is suggested that you look over the exam first to get an idea of how to apportion your time.

Please be careful.

1. [10 points]

Derive a regular expression for the set of all strings of 0's and 1's representing a multiple of 3 as a binary numeral.

Hint: The following DFA was shown in the text to accept this set of strings:



2. [20 points]

For each language below:

- If the language is regular, construct the minimal DFA for it.
- If the language is not regular, describe the abstract states of the language and how they are connected.

a. The set of all strings in {0, 1}* other than 101.

b. The set of all strings in {0, 1}* ending in 101.

c. The set $\{x\ 0\ x\ \mid x \in \{1\}^*\}$.

3. [40 points]

For each language below:

a. If the language is regular, describe the operation of a DFA or NFA accepting the language.

b. If the language is not regular, but is context-free, describe the operation of a (possibly non-deterministic) pushdown acceptor accepting the language.

c. If the language is not context-free, give a convincing argument for this fact.

Languages (x^r designates the reverse of string x and $|x|$ is the length of x.):

a. $A = \{x\ x^r \mid x \in \{0, 1\}^*\}^*$ [note the outermost asterate]

b. $B = \{x\ x^r\ x\ x^r \mid x \in \{0, 1\}^*\}$

c. $C = \{x\ y\ y^r \mid x \in \{0, 1\}^*, y \in \{0, 1\}^*, |x| < 1000\}$

d. $D = \{x\ y\ y^r \mid x \in \{0, 1\}^*, y \in \{0, 1\}^*, |y| < 1000\}$

4. [30 points]

For each logical expression below:

a. Determine whether or not the expression is a tautology using any method (including inspection, but be careful).

b. If it is a tautology, prove it using natural deduction.

c. If it is not a tautology, give a counterexample (in the form of a valuation).

E: $((p \wedge r) \rightarrow (q \wedge s)) \rightarrow ((p \rightarrow q) \wedge (r \rightarrow s))$

F: $((p \rightarrow q) \wedge (r \rightarrow s)) \rightarrow ((p \wedge r) \rightarrow (q \wedge s))$

G: $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$

H : $(p \rightarrow q) \vee (q \rightarrow r)$