Competitive Learning
Some Types of Learning

- **Supervised learning**: training using desired response for given stimuli ("rote" learning)
- **Unsupervised learning**: classification by "clustering" of stimuli, without specified response
- **Hybrid**: e.g. unsupervised to form cluster, supervised to learn desired response to class
Competitive Learning

- A form of **unsupervised** learning, but **combinable with supervised** learning.

- Neurons “compete” based on proximity to input pattern.

- Neuron closest to pattern (the “**winner**”) adjusts its weight to be still closer.
2-way competition

Presented input pattern

Input presentation carries the assumption that the network is supposed to “learn” the input.

The “winner”
2-way competition

Presented input pattern

Neurons

The “winner” becomes more like the input.

The “loser” stays as is.

Input presentation carries the assumption that the network is supposed to “learn” the input.
Why not make the winner *exactly* like the input?

- There may be many more distinct input patterns than neurons.
- By “averaging” its behavior, a neuron can put a large number of distinct, but similar inputs into the same category.
Categorizing Inputs by 2 neurons
An Application

- Displaying an image file with “millions of colors” using a palate of, say, 256 colors.

- Each color in the image has to be mapped into one of the colors in the palate.

- Map each image color into the closest one of the 256 palate colors.

- In this case, a **competitive network** can learn a reasonable set of colors to use for a given image.
Related Applications

- Use the reduction in number of colors of the image to **store** a version of the image more **compactly** (1M color $\rightarrow$ 256 colors reduces the number of bits by a factor of 2.5),

or to **transmit** the version image over a slow channel.
A Competitive Neural Network

- when presented with patterns from the same selection of inputs repeatedly, will tend to stabilize so that its neurons are representatives of clusters of closer inputs.

- Each neuron will tend to be similar to inputs in its cluster (like a chameleon, perhaps)
Measures of similarity or closeness (opposite: dissimilarity or distance)

- Suppose $x$ is an input vector and $w_i$ the weight vector of the $i^{th}$ neuron.

- One measure of distance is the **Euclidean distance**:
  \[
  \| x - w_i \| = \sqrt{\sum_i (x_j - w_{ij})^2} \\
  = \sqrt{(x_j - w_{ij})^T(x_j - w_{ij})}
  \]
  (vector inner product)
Measures of distance

- The discrete metric:

\[ d(x, w_i) = 0 \text{ if } x = w_i, \ 1 \text{ otherwise} \]
Measures of distance

- Another measure of distance, used when the values are integer, is the “Manhattan” “city-block”, or “taxi cab” distance:

$$\| x - w_i \| = \sum_i (|x_j - w_{ij}|)$$
Measures of distance

Another measure of distance, used when the values are 2-valued, is the “Hamming distance”:

\[ \text{sum}_i( |x_j = w_{ij}| ) \]

0 when the values are equal, 1 otherwise
Richard Hamming (1915-1998)
Properties of Distance (i.e. a Metric)

- \( d: P \times P \rightarrow R \)
  where \( P \) is the space of patterns and
  \( R \) is the set of real numbers

- \((\forall x, y) \ d(x, y) > 0\)

- \((\forall x) \ d(x, y) = 0 \text{ iff } x = y\)

- \((\forall x, y) \ d(x, y) = d(y, x)\)

- \((\forall x, y, z) \ d(x, z) \leq d(x, y) + d(y, z)\)
Example for Different Metrics

- Suppose $x = [1 \ 1 \ -1 \ 1]$, $w = [1 \ -1 \ -1 \ -1]$

- Euclidean distance $= \sqrt{0^2 + 2^2 + 0^2 + 2^2} = 2.83...$

- Discrete metric $= 1$

- Manhattan distance $= 0 + 2 + 0 + 2 = 4$

- Hamming distance $= 0 + 1 + 0 + 1 = 2$
A measure of similarity is given by the inner product

The inner product

\[ x \cdot w_i \]

is larger when \( x \) is “closer to” \( w_i \).

Therefore inner product is not a metric.

Usually it is best if \( x \) and \( w_i \) are normalized before using this measure:

\[ \| x \| = \| w_i \| = 1 \]

In this case, \( (1 - xw_i)/2n \) is a distance, where \( n \) is the dimension (check me on this).
Inner Product is not a Metric

- \([1 1 -1 1] \times [1 1 -1 1] = 4\)
- \([1 1 -1 1] \times [-1 -1 1 -1] = -4\)
- \([1 1 -1 1] \times [1 -1 -1 -1] = 0\)
Inner product as cosine

- When the pattern space consists of vectors of numbers, the normalized inner product is the cosine of the angle between \( x \) and \( w_i \) as vectors.
Determining a Winner

- The winner is the neuron with weight either:
  - the smallest distance to the input, or
  - the largest inner product with the input.

- Again, if inner products are used, it is best to **normalize** the weight and input first, or use only normalized values.
Example: Hamming Network (competitive, non-learning)

- Input neuron weight vectors = stored patterns
- Inner products
- “1-hot” code
- Winner indicator

The diagram shows a network with input neurons connected to weight vectors $w_1, w_2, w_3, \ldots, w_n$. The max network selects the maximum inner product, resulting in a “1-hot” code.
“Neural” Implementation of a Max Sub-Network

- a recurrent neural net that cycles values through neurons, eliminating one loser each cycle until only the winner is left.

- (This is totally unnecessary from a strictly computational viewpoint.)

- Each neuron has as inputs the outputs of all neurons including itself.

- Self-weights are 1; Weights from other neurons are $-\varepsilon$, where $\varepsilon$ is any quantity $< 1/($# of neurons$)$. 
Max Network

- Activation functions are “poslin”:

  \[ \text{poslin}(x) = x \text{ if } x > 0, \ 0 \text{ otherwise} \]

- The network is operated \textit{synchronously}.
- The initial outputs are forced to those of the input values.
- On each cycle, each neuron computes \text{poslin}(\text{weighted inputs}).
For the $i^{th}$ neuron

$$y_i := \text{poslin}(y_i - \epsilon \sum y_j)_{j \neq i}$$

$$= (1+\epsilon)y_i - \epsilon \sum y_j$$

These weights are designed so that:

- all but one output is non-zero after $n$ cycles (assuming inputs were originally distinct)
- all outputs persist at the same value after $n$ cycles
MaxNet Example

- $n = 4$ neurons, take $\varepsilon = 0.2 < 1/4$

$$y_i := (1+\varepsilon)y_i - \varepsilon \sum y_j$$

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<th>step</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
<th>sum</th>
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</tbody>
</table>
Matlab *compet* function  
(non-learning)

COMPET(N) takes one input argument,  
N - SxQ matrix of net input (column) vectors.  
and returns output vectors **with 1 where each net input  
vector has its maximum value, and 0 elsewhere.**

```
compet([-3; -1; 5; 2; -9]) ==> (3,1) 1
= compet(Wp)
```

**diagram**

```
\[ \text{Input} \rightarrow \text{Competitive Layer} \]
\[ p_{Rx1} \rightarrow W_{SxR} \rightarrow n_{Sx1} \rightarrow C_{Sx1} \rightarrow a_{Sx1} \]

\[ R \rightarrow S \]
```

\[ a = \text{compet}(n) \]
\[ = \text{compet}(Wp) \]
Using Competition in Conjunction with Learning

- Input presented
- Winner selected
- The winner learns

- Others “close to” winner may learn as well.
Instar Rule (Stephen Grossberg)

\[ i^w(q) = i^w(q - 1) + \alpha a_i(q)(p(q) - i^w(q - 1)) \]

1 for \( i = \text{winner} \)
0 otherwise

Pattern - weight

Learning rate

Only winner learns
Graphical Representation

\[ w_i*q = w_i*q_1 - \alpha*p*q (1 - \alpha) w_i*q_1 - \alpha*p(q) \]
Graphical Representation

\[ i_w(q) = i_w(q-1) + \alpha(p(q) - i_w(q-1)) \]

\[ i_w(q) = (1 - \alpha) i_w(q-1) + \alpha p(q) \]
Matlab Demos

- nnd14cl (competitive learning)
- A vector input is chosen on the circumference of a circle.
- A competition is held among the neurons.
- The closest neuron learns by moving closer to the input.
- Other neurons stay put.
Matlab Demos

- democ1
- initial display:

Data points (red) vs. 2D weights of neurons (coinciding initially)
The neurons are now better *representatives* of the data.
Competitive Learning vs. Clustering

- The preceding example shows that competitive learning is one way to achieve clustering.
- We discussed the **k-means clustering** algorithm earlier for example.
- In k-means, the computation of new centers can be viewed as a kind of competition, since the new centers are computed by summing the inputs **closest** to a given old center.
Possible Instability

If the input vectors don’t fall into nice clusters, then for large learning rates the presentation of each input vector may modify the configuration so that the system will undergo continual evolution.
Possible Instability

If the input vectors don’t fall into nice clusters, then for large learning rates the presentation of each input vector may modify the configuration so that the system will undergo continual evolution.

Solution: Gradually decrease the learning rate ("annealing").
Another problem with competitive learning is that neurons with initial weights far from any input vector may never win and thus become useless.
Have a Heart

Solution: Add a negative bias to each neuron, and increase the magnitude of the bias as the neuron wins. This will make it harder for a neuron to win if it has won often.

This is called the “conscience” method.
Competitive Learning with Explicit Output Classification

Learning Vector-Quantization

Counterpropagation
Vector Quantization

- VQ is a technique used to reduce the dimensionality of data.
- The original data is a set of vectors, of dimension $n$, say.
- The vectors are mapped into a set of smaller dimension, $m$, of codebook values.
- The codebook values are used for storage or transmission (some information content is lost).
- The codebook values, upon receipt or retrieval, are re-translated into values close to the original data values (will not be exact).
Codebook

codebook values

close to a subset of original

many-one

one-one
inverse-image = some kind of clustering

codebook values

close to a subset of original
Voronoi Regions

Typically codebook code representatives are centers of Voronoi Regions in the data space. Voronoi regions are sets of points closest to one representative vs. another. Below each polygon is a Voronoi region, with representatives as dots.

closest $\Rightarrow$ error is minimized
Chicken vs. Egg

- Which comes first, the Voronoi regions or the representatives?

- Initially we just have a set of points with neither.

- We specify the maximum size of the codebook (assume the number of points is larger).

- Then use a clustering technique, such as **k-means clustering**.
k-means clustering in more detail

- Choose an initial set of representatives by some method.
- Iterate the following:
  - For each point, find its closest representative; this determines a set of Voronoi regions.
  - Compute the centroids of the regions and use them as the new representatives.
- Until the desired error bound is reached.
- Note: The representatives need not be actual data points.
Termination Condition

• Compute $\text{MSE} = \Sigma \{\text{distance-from-rep}(p)^2 \mid p \text{ in region}\}$
  \#regions

  where the sum is over all regions.

• We want MSE to be below a certain bound.
Computational Cost

- $O(n^2)$ per iteration, where $n$ is the number of points.
Other Clustering Methods

• There are many.
• For a survey, see this tutorial: http://www.pitt.edu/~csna/reports/janowitz.pdf which was listed on the website of the:
  
  Classification Society of North America
• k-means clustering is mentioned on page 30.
Learning Vector-Quantization (LVQ)
Learning Vector Quantization (LVQ)

- Combine **competitive** learning with **supervision**.
- Competitive learning achieves clusters.
- Assign a **class** (or output value) to each cluster.
- Reinforce cluster representative (a neuron) when it **classifies** input in the **desired** class:
  - **Positive reinforcement**: pull the neuron weights toward the input.
  - **Negative reinforcement**: push the weights away.
Learning Vector Quantization

Training examples:

\[ \{p_1, t_1\}, \{p_2, t_2\}, \ldots, \{p_Q, t_Q\} \]

If the input pattern is classified \textit{correctly}, then move the \textit{winning} weight toward the input vector according to the rule:

\[
i^*w^1(q) = i^*w^1(q-1) + \alpha (p(q) - i^*w^1(q-1)) \quad a_{k^*}^2 = t_{k^*} = 1
\]

If the input pattern is classified \textit{incorrectly}, then move the \textit{winning} weight \textit{away} from the input vector:

\[
i^*w^1(q) = i^*w^1(q-1) - \alpha (p(q) - i^*w^1(q-1)) \quad a_{k^*}^2 = 1 \neq t_{k^*} = 0
\]
LVQ Rules

Input pattern correctly classified

Input pattern incorrectly classified

input

neurons

input

neurons
Learning Vector Quantization
Second Layer

• The classification mapping from selection of cluster representative to output can be accomplished by a second layer, following the competitive layer.

• This second layer is essentially a matrix multiply, so can be represented by neural weights as usual. These weights may be pre-assigned and fixed.
Example patterns with targets:

\[
\begin{align*}
\{ & \mathbf{p}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{t}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \} & \{ & \mathbf{p}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{t}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \} & \{ & \mathbf{p}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{t}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \} & \{ & \mathbf{p}_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{t}_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \} \\
\end{align*}
\]

Competitive layer weights:

\[
W^1(0) = \begin{bmatrix} (1w^1)^T \\ (2w^1)^T \\ (3w^1)^T \\ (4w^1)^T \end{bmatrix} = \begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 0.75 \\ 1 & 0.25 \\ 0.5 & 0.25 \end{bmatrix}
\]

Classification layer weight (fixed):

\[
W^2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}
\]
Iteration 1, first layer

\[ a^1 = \text{compet}(n^1) = \text{compet} \begin{pmatrix} -\|w^1_1 - p_1\| \\ -\|w^1_2 - p_1\| \\ -\|w^1_3 - p_1\| \\ -\|w^1_4 - p_1\| \end{pmatrix} \]

\[ a^1 = \text{compet} \begin{pmatrix} -\left\| \begin{bmatrix} 0.25 & 0.75 \end{bmatrix}^T - \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\| \\ -\left\| \begin{bmatrix} 0.75 & 0.75 \end{bmatrix}^T - \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\| \\ -\left\| \begin{bmatrix} 1.00 & 0.25 \end{bmatrix}^T - \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\| \\ -\left\| \begin{bmatrix} 0.50 & 0.25 \end{bmatrix}^T - \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\| \end{pmatrix} = \text{compet} \begin{pmatrix} -0.354 \\ -0.791 \\ -1.25 \\ -0.901 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \]
Iteration 1, second layer

\[ a^2 = W^2 a^1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

This is the correct class, therefore the weight vector is moved toward the input vector.

\[ w^1_1(1) = w^1_1(0) + \alpha(p_1 - w^1_1(0)) \]

\[ w^1_1(1) = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} + 0.5 \left( \begin{bmatrix} 1 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 0.125 \\ 0.75 \end{bmatrix} \right) = \begin{bmatrix} 0.125 \\ 0.875 \end{bmatrix} \]
Iteration 1 illustrated

Input presented

Winner positively reinforced
Final Outcome after many iterations
Matlab LVQ demos

• nnd14lv1, nnd14lv2

Blue vs. black represent class of neuron, desired class of input sample.

Selected input sample changes to green if correctly classified, red if not; weights adjusted.
Variant: LVQ2

- If the winning neuron in the hidden layer correctly classifies the current input, the response is the same as in LVQ1.

- If the winning neuron in the hidden layer incorrectly classifies the current input, we move its weight vector away from the input vector, as in LVQ, and

- also move the weights of the closest neuron to the input vector that would have correctly classified the input toward the input vector.
LVQ2 Rules

Input pattern correctly classified

Input pattern incorrectly classified

neurons

input

neurons

input
Counterpropagation Networks
Counterpropagation Networks

- Invented by Robert Hecht-Nielson, founder of HNC Inc., recently acquired by Fair Isaac Co., Inc.

- Consists of two opposing networks, one for learning a function, the other for learning its inverse.

- Each network has two layers:
  - A first layer that clusters inputs (sometimes stated to be a self-organizing map, see later).
  - An “outstar” second layer to provide the output values for each cluster.
Outstar and Instar
(due to Stephen Grossberg)

- An instar responds to a single input vector.
- An outstar produces a single output vector when stimulated with a binary value.
Biologically, outstar would be synaptic weights, while instar would have *dendritic* ones. However, it is common to refer to both weights as “synaptic”.

\[
\text{Output} = wx
\]
Simplified Instar and Outstar

- **Outstar learning rule:**
  \[ \Delta w = \alpha x (d - w) \]
  \( (d = \text{desired}) \)

- **Instar learning rule:**
  \[ \Delta w = \alpha (x - w) d \]
  \( (d = \text{desired}) \)

**Diagram:**
- **Outstar:**
  - Input: \( x \), \( e.g. \in \{0, 1\} \)
  - Weights: \( w_1, w_2, w_3, \ldots, w_n \)
  - Output: \( wx \)

- **Instar:**
  - Input: \( x_1, x_2, x_3, \ldots, x_n \)
  - Weights: \( w_1, w_2, w_3, \ldots, w_n \)
  - Output: \( wx \)

Input and output values are reals.
Outstar and Instar

- Variations are possible, e.g. weight-decay.
Counterpropagation Operation

• An outstar neuron is associated with each cluster representative.

• Given an input, the winner is found.

• The outstar is then stimulated to give the output.
Counterpropagation Notes

• Since these networks operate by recognizing input patterns in the first layer, one would generally use lots of neurons in this layer.

• There is a certain similarity to the approach of a radial-basis function network.

• In a full counterpropagation network, if the relationship being learned is a function, then inverse of the function can be learned at the same time.
Counterpropagation Network Computing Numeric Reciprocal

Figure 4.44 Full counterpropagation network for $y = 1/x$.

Competitive layer; only winner fires.
Outstar weights determine output.
Counterpropagation vs. LVQ

- Both combine Supervise and Unsupervised learning.
- LVQ is used for classification; Counterprop can be used for function computation.
- LVQ has one training phase; Counterprop has two.
- LVQ output weights are fixed throughout; Counterprop weights are trained in a second phase.
Counterpropagation Applets
http://eple.hib.no/~hib00ssl/BitMapRecognizerCounterpropagation.html
Counterpropagation Applications

- TeleSpec: IR Spectrum of Molecular Structures
  www2.chemie.uni-erlangen.de/services/telespec/english/StructureInput.html

- A trained neural network is able to predict an infrared spectrum for a molecule \textit{it has not seen before}.

- The structure code of the query structure is compared with the structure block of the neural network.

- The similarity criterion is the MSE between the query structure code and the neuron weights.

- The most similar neuron, the winning neuron, acts as a pointer to the corresponding information in the output block. The output block provides the simulated spectrum.
TeleSpec Operation

- The first step in simulating an IR spectrum is the input of a query structure. TeleSpec provides an integrated Java-based structure editor that converts a graphical input into a so-called SMILES string. SMILES is a standardized ASCII description of the connectivity of a molecule.

- The SMILES string is then converted into 3D coordinates of the atoms with the integrated structure generator CORINA, (COoRdINAtes), a rule and data based system, that automatically generates three-dimensional atomic coordinates from the constitution of a molecule as expressed by a connection table or linear code, and which is powerful and reliable to convert large databases of several hundreds of thousand or even millions of compounds.

http://www2.chemie.uni-erlangen.de/software/corina/index.html
TeleSpec Operation

- In addition to the three-dimensional arrangement of the atoms in a molecule, physicochemical properties are needed to describe molecules in more detail. This is achieved by integrating a series of empirical methods for the calculation of electronic and energy effects, such as charge distribution or polarizability effects. This program package is called PETRA (Parameter Estimation for the Treatment of Reactivity Applications).
- This is followed by a structure coding step (shown as ARC) which encodes into a numeric vector using either of two methods.
TeleSpec use of Counterpropagation
TeleSpec Network Training

- The network learns the relationship between structure and spectrum by analyzing a set of pairs of molecules and spectra in a training phase. For each data point, a pair of structure and spectrum, two steps are performed:
  - **Determining the most similar neuron** For each data point, the most similar neuron is determined by calculating the rms error between the structure code (red) of the training structure and the input (upper) block of the neural network.
  - **Adjustment of the neuron weights** The weights of the winning neuron are adjusted to become more similar to the training data point. The weights of the neighbor neurons are also adjusted with the rate of adjustment decreasing with increasing distance from the winning neuron. The adjustment is applied to the structure (green) block and the spectrum (red) block of the network. This training step establishes the correlation between structures and spectra and stores it in the network.
- A trained counterpropagation network acts as an interpolator and is able to predict a spectrum for a molecule it has never seen before.
Another Counterpropagation Application: Dolphin Echolocation


- Describes preliminary work on using a counterpropagation artificial neural network to recognize echoes from objects ensonified in a test pool by an artificial dolphin click and in Kaneohe Bay by a dolphin during performance of a delayed matching-to-sample task. Selected echoes were analyzed and successfully recognized by the network. Target recognition abilities of an echolocating dolphin and the neural network were also compared. In a noisy natural environment, the dolphin was 94.5 percent correct and the network was 96.7 percent correct. Possible applications of neural networks to echolocation studies are discussed.
Robocup soccer application “a4ty” (dssg.cs.rtu.lv/download/robocup/a4ty2003_long.pdf)

“We applied [counterpropagation] for pass selection. An agent trained with offline learning needs to decide whether it should give a pass to a partner or not. If it makes an incorrect pass (the ball is intercepted by an opponent) in online mode in the given situation, the network updates weights so as next time the probability of choosing this action for the same (or similar) situation would be lower.

However, this approach has not been implemented in the final version of a4ty team since the overall performance (approximation capabilities) of counterpropagation network is worse than, for example, multilayer perceptron has. Thus, counterpropagation neural network has been replaced by multilayer perceptron (MLP)”.