1. Construct a DFA that recognizes each of the following languages. Justify your answer by describing, in English, the purpose of each state.
   a. The strings over \{p,q,r,\neg,\land,\lor,(,)\} that represent well formed formula in conjunctive normal form (CNF).
   b. The strings over \{a,b\} in which the parities of a and b are the same; i.e. a string in the language has an even number of both a and b or an odd number of both a and b.
   c. The strings over \{a,b\} in which each consecutive block of 5 symbols has at least two b’s; i.e. if \(s_0s_1s_2s_3s_4\) is in the language then for any \(i, 0 \leq i \leq n-4\), the substring \(s_is_{i+1}s_{i+2}s_{i+3}s_{i+4}\) contains at least 2 b’s.

2. Construct an NFA that recognizes the following languages. Justify your answer by describing, in English, the purpose of each state.
   a. The strings over \{a,b\} containing at least 2 a’s and such that some pair of a’s is separated by a string whose length is a multiple of 3.
   b. The strings over \{a,b,c\} that have the same value when multiplied from the left as from the right according to the following (non-associative) multiplication table:
      \[
      \begin{array}{c|ccc}
      & a & b & c \\
      \hline
      a & a & a & c \\
      b & c & a & b \\
      c & b & c & a \\
      \end{array}
      \]
      For example, aba is in the language because multiplication from the left yields \((ab)a=aa=a\) and from the right yields \(a(ab)=aa=a\). But bac is not because \((ba)c=cc=a\) but \(b(ac)=bc=b\).

3. Construct a regular expression for each of the following languages. Provide justification of correctness.
   a. The set of strings over \{0,1\} that represent, in binary, a number that is equivalent to zero modulo 3.
   b. The strings over \{a,b\} with an equal number of a’s and b’s such that in every prefix the number of a’s and the numbers of b’s differs by at most 2; i.e. if \(s_0s_1\ldots s_n\) is in the language then for any \(i, 0 \leq i \leq n\), the prefix \(s_0s_1\ldots s_i\) has the property that the number of a’s and b’s differ by at most 2.