1. Convert the following NFA to a DFA using the algorithm given in class.

2. Convert the NFA in problem 1 into a regular expression using the algorithm described in class. Show your work.

3. Prove the following improved pumping lemma. Let $L$ be any regular language. Then there exists an $n$ such that for any $x$ in $L$, for any way of writing $x$ as $z_1z_2z_3$ with $|z_2| \geq n$, there exists strings $w_1$, $w_2$, and $y$ such that $w_1yw_2z_3$, $|w_1y| \leq n$, $|y| > 0$, and the string $z_1w_1y^kw_2z_3$ is in $L$ for all $k \geq 0$.

4. Which of the following languages over $A=\{a,b\}$ are regular? Prove your answer. (Hint: You may find the improved pumping lemma particularly helpful for at least one of these problems.)

   a. $\{a^{3n} | n \geq 1\}$
   b. $\{a^ib^k | i>j>k\}$
   c. $\{a^ib^k | i+j+k \text{ is even}\}$
   d. $\{xx^Rw | x\neq\epsilon\}$ where $x^R$ denotes the string $x$ in reverse order
   e. $\{x | x=x^R\}$
   f. $\{xx^Rw | x\neq\epsilon\}$
5. Let \( L \) be a regular language. Which of the following are also regular? Prove your answer.
   
   a. \( L^R = \{ x \mid x^R \in L \} \)
   
   b. \( \frac{1}{2} L = \{ x \mid \exists y \ | y| = |x| \text{ and } xy \in L \} \)
   
   c. \( \{ x \mid xx^R \in L \} \)
   
   d. \( \{ xz \mid \exists y \ | y| = |x| = |z| \text{ and } xyz \in L \} \)