Scheduling Anomalies
Deterministic Scheduling

- Assume times of tasks (and communication) are known, which they often aren’t.
- Problem is NP-hard for all but the most trivial classes of assumptions.
- Unexpected scheduling anomalies.
The following are expected to reduce overall execution time:

- Reducing execution times of individual tasks
- Relaxing precedence constraints between tasks
- Adding more processors
The following are expected to reduce overall execution time:

- Reducing execution times of individual tasks
- Relaxing precedence constraints between tasks
- Adding more processors

For some algorithms, these can actually increase the execution time.
Priority Scheduling Anomalies

Priority number (lowest number = highest priority)

Execution time

precedence constraints

$T_1/3 \rightarrow T_9/9$

$T_2/2$

$T_3/2$

$T_4/2 \rightarrow T_5/4$

$\rightarrow T_6/4$

$\rightarrow T_7/4$

$\rightarrow T_8/4$

$T_i/\tau$
Consider Scheduling on 3 processors

\[ \frac{T_1}{3} \]
\[ \frac{T_2}{2} \]
\[ \frac{T_3}{2} \]

\[ \frac{T_4}{2} \rightarrow \frac{T_5}{4} \]
\[ \frac{T_6}{4} \]
\[ \frac{T_7}{4} \]
\[ \frac{T_8}{4} \]

\[ \frac{T_9}{9} \]
Consider Scheduling on 3 processors

\[
\begin{array}{c}
\frac{T_1}{3} & \quad \frac{T_9}{9} \\
\frac{T_2}{2} & \quad \frac{T_4}{2} \\
\frac{T_3}{2} & \\
\end{array}
\]
Consider Scheduling on 3 processors

\[
\begin{array}{cccc}
T_1/3 & & T_9/9 \\
T_2/2 & T_4/2 & T_5/4 & T_7/4 \\
T_3/2 & & T_6/4 & T_8/4 \\
\end{array}
\]

Total time = 12
Consider Scheduling on 4 processors

\[ T_1/3 \]
\[ T_2/2 \]
\[ T_3/2 \]
\[ T_4/2 \]
Consider Scheduling on 4 processors

\[ \frac{T_1}{3}, \frac{T_8}{4} \]

\[ \frac{T_2}{2}, \frac{T_5}{4} \]

\[ \frac{T_3}{2}, \frac{T_6}{4} \]

\[ \frac{T_4}{2}, \frac{T_7}{4} \]
Consider Scheduling on 4 processors

<table>
<thead>
<tr>
<th>$T_1/3$</th>
<th>$T_8/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_2/2$</td>
<td>$T_5/4$</td>
</tr>
<tr>
<td>$T_3/2$</td>
<td>$T_6/4$</td>
</tr>
<tr>
<td>$T_4/2$</td>
<td>$T_7/4$</td>
</tr>
</tbody>
</table>

Total time = 15
Consider Relaxing Constraints

\[ \frac{T_1}{3} \rightarrow \frac{T_9}{9} \]
\[ \frac{T_2}{2} \]
\[ \frac{T_3}{2} \]
\[ \frac{T_4}{2} \rightarrow \frac{T_5}{4} \]
\[ \frac{T_5}{4} \rightarrow \frac{T_6}{4} \rightarrow \frac{T_7}{4} \rightarrow \frac{T_8}{4} \]

Remove
Consider the relaxed constraints on 3 processors

\[
\begin{array}{ccc}
\frac{T_1}{3} & \frac{T_8}{4} & \frac{T_9}{9} \\
\frac{T_2}{2} & \frac{T_4}{2} & \frac{T_5}{4} \\
\frac{T_3}{2} & \frac{T_7}{4} & \frac{T_6}{4} \\
\end{array}
\]

Total time = 16
Cause of Anomalies

- Obviously the anomalies are caused by the use of the priority rule in scheduling:
  - This rule is cheap to implement (O(n)).
  - It does not take into account optimizations that would be possible by violating strict priority.

- In general, finding true optimum would entail a search, which tends to be much more expensive.
Bounds on Anomalies
(due to R.L. Graham)

- Let ‘ designate times for system with relaxed constraints and shorter individual times. Then
  \[ \frac{\text{Time}(m)}{\text{Time}(m')} \leq 1 + \frac{(m-1)}{m'}, \]
  where \( m' \geq m \) are numbers of processors.

- Example: \( \frac{\text{Time}(2)}{\text{Time}(3)} \leq \frac{4}{3} \).
- Worst case: \( \frac{\text{Time}(m)}{\text{Time}(m')} < 2 \).