Algorithms
Computer Science 140 & Mathematics 168
Spring 2009
Homework 11b
Due Tuesday, April 21

1. [**20 Points**] **Tally Circuits!** A tally circuit has n binary inputs and $\lceil \log_2(n+1) \rceil$ outputs. Interpreted as a binary number, the outputs give the number of 1's in the input. For example, if the input is 1101111 then the output is 110, indicating that there are six 1's in the input. Describe a tally circuit with depth $O(\log n)$ and size $O(n)$. Show your analyses in detail. (Careful here – achieving the depth and size requirements will require some care.)

2. [**20 Points**] **Majority Circuits!** A majority circuit with n boolean inputs $x_1, x_2, \ldots, x_n$ outputs a 1 if $x_1 + x_2 + \ldots + x_n > n/2$ and outputs 0 otherwise. Describe a circuit for an n-input majority function that has depth $O(\lg n)$. Describe and analyze your circuit carefully. What is the size of your circuit?

3. [**20 Points Points**] **More fun with Norelco Shavers!** Give efficient EREW algorithms to compute the preorder, inorder, and postorder numberings for an arbitrary binary tree with n nodes. For each of the three parts of this problem (preorder, inorder, and postorder), describe the values that must be placed in processors A, B, C, explain where the solution is found, and briefly explain why your algorithm works.

4. [**15 Points Points**] **Parallel Matrix Multiplication!** In this problem we'll investigate the problem of finding the product of two $n \times n$ matrices using a shared-memory parallel computer with n^3 processors.

   (a) Show that this can be done in $O(\lg n)$ time if the machine has CREW. Is this algorithm work-efficient? No!

   (b) Now describe the fastest algorithm you can find for this problem that operates on a machine with EREW.

5. [**15 Points**] **Sorting in Parallel!** Give a parallel algorithm for sorting n numbers in $O(\log n)$ time using n^2 processors. What model (EREW, CREW, or CRCW) does your algorithm require? (Use the "weakest" model that you can! EREW is weaker than CREW is weaker than CRCW.)