

CS81 Assignment 2
Due Monday, 3 February 2009

1. Prove the following sequents using natural deduction. If possible, use only constructive rules, reserving other rules for when it seems necessary. Give both a tabulation and a tree format proof for the first 3 examples. For the remainder, use the format of your choice. (You may use JAPE to format your tabulations, if desired.)

- a. $\vdash \neg p \rightarrow (p \rightarrow (p \rightarrow q))$
- b. $p \vee q, \neg q \vee r \vdash p \vee r$
- c. $(p \rightarrow r) \wedge (q \rightarrow r) \vdash (p \vee q) \rightarrow r$
- d. $(p \vee q) \rightarrow r \vdash (p \rightarrow r) \wedge (q \rightarrow r)$
- e. $\neg p \wedge \neg q \vdash \neg(p \vee q)$
- f. $\neg(p \wedge q) \vdash \neg p \vee \neg q$
- g. $\vdash (p \rightarrow q) \vee (q \rightarrow r)$

2. Give tableau proofs for sequents a, d, f, g above.

3. Use a tableau to find the interpretations that show that the following is not valid: $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow r)$.

4. Show that, given only **one** of the rules RAA (reductio ad absurdum), LEM (law of the excluded middle), DNE (double not elimination), the other two rules can be derived using only constructive rules in addition.