CS81 **Assignment 3**
Due Monday, 10 February 2009

In Ben-Ari, Exercises 5.9, page 125. You may use JAPE for constructing your proofs. (Note that, in JAPE, if the non-empty universe assumption is needed, you need to add 'actual i' as a premise, to indicate there is at least one actual element. This gives you an element to use for ∃ introduction.)

1. Exercise 1.

2. Exercise 2:
   Prove only the third formula informally, by arguing with respect to an arbitrary interpretation.

3. Exercise 2:
   Prove all 4 formulas by natural deduction.

4. Exercise 4

1. Find an interpretation which falsifies $\exists x p(x) \rightarrow p(a)$.

2. Prove that (some of) the formulas in Figure 5.2 are valid, especially

$$\exists x(A(x) \rightarrow B(x)) \leftrightarrow (\forall x A(x) \rightarrow \exists x B(x)),$$
$$(\exists x A(x) \rightarrow \forall x B(x)) \rightarrow \forall x(A(x) \rightarrow B(x)),$$
$$\forall x(A(x) \vee B(x)) \rightarrow (\forall x A(x) \vee \exists x B(x)),$$
$$\forall x(A(x) \rightarrow B(x)) \rightarrow (\exists x A(x) \rightarrow \exists x B(x)).$$

4. For each of the following formulas, either prove that it is valid or give a falsifying interpretation.

$$\exists x \forall y((p(x, y) \wedge \neg p(y, x)) \rightarrow (p(x, x) \leftrightarrow p(y, y))),$$
$$\forall x \forall y \forall z(p(x, x) \wedge (p(x, z) \rightarrow (p(x, y) \vee p(y, z)))) \rightarrow \exists y \forall z p(y, z).$$