CS81 **Assignment 5**
Meta-Proofs, and More
Due Monday, 25 February 2009

Assume for context the lectures of the week of 17 February, and the accompanying slides, which are posted on the web.

1. [5] Prove that, for any formulas φ and ψ, $\varphi \models \psi$ implies $\models \varphi \rightarrow \psi$, using reasoning about interpretations (related to slide 20).

2. [5] Prove that, for any formulas φ and ψ, $\vdash \varphi \rightarrow \psi$ implies $\varphi \vdash \psi$ with respect to natural deduction rules (related to slide 20).

3. [15] Prove the part of the structural induction step of propositional soundness for the $\rightarrow$E rule. (see slide 14).

4. [20] Prove the part of the structural induction step of propositional soundness for the $\vee$E rule. (see slide 15).

5. [20] Prove the part of the structural induction on formulas step for the completeness theorem, where η is of form $\rho_1 \rightarrow \rho_2$ (see slide 28). Show any natural deduction steps that you claim exist.

6. [Extra Credit, 40] Show the natural deduction proof that would be generated, in the proof of the completeness theorem, corresponding to:

   $\models (\neg p \wedge q) \rightarrow (q \wedge \neg p)$

7. (Not meta-proofs) Prove, using the sequent calculus [7 each]:
   a. $\vdash P \vee \neg P$
   b. $\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$
   c. $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$
   d. $\vdash (R \rightarrow S) \rightarrow ((P \rightarrow R) \rightarrow (P \rightarrow S))$
   e. $(P \rightarrow Q) \wedge (Q \rightarrow R), \neg R \vdash \neg P$

You may use JAPE, but please know how to do by hand.