

Computer Science 81, Spring 2009

Assignment 6

Due Mon. March 2

Checking your solutions with Otter-lambda or Prover9 is advisable, but work the problems for yourself first.

1. [10 points] Establish the satisfiability or unsatisfiability of the following sets of clauses by the resolution method. If a set is satisfiable, give an assignment that shows this.
 - a. $\{p \vee q, p \vee \neg r, \neg p \vee r, \neg p \vee \neg q, \neg q \vee r, q \vee \neg r\}$
 - b. $\{p \vee q, \neg p \vee r, \neg p \vee \neg q, \neg q \vee r, q \vee \neg r\}$
2. [10 points] Determine whether each pair of terms is unifiable, and if so, give the most general unifier. Note: u, v, w, x, y, z are variables; a, b, c, d are constants.
 - a. $p(f(x, g(y)), y)$ vs. $p(f(g(a), z), b)$
 - b. $p(x, f(y, a), y)$ vs. $p(f(a, b), v, z)$
 - c. $p(x, f(x))$ vs. $p(g(a), f(h(a)))$
3. [20 points] Establish the satisfiability or unsatisfiability of the following sets of clauses by resolution. If a set is satisfiable, give an interpretation that shows this.
 - a. $\{p(x), q(y, a) \vee \neg p(y), \neg q(b, a)\}$
 - b. $\{p(a), q(y, a) \vee \neg p(a), q(b, x)\}$
 - c. $\{q(x), \neg p(y) \vee \neg p(g(a)) \vee \neg q(a), p(z) \vee \neg q(w)\}$
4. [20 points] Transform each formula into an equivalent set of clauses.
 - a. $\exists y \forall x (p(x, y) \rightarrow q(x))$
 - b. $\forall x \exists y (p(x, y) \vee \exists z q(x, y, z))$
5. [40 points] Prove each formula by the resolution method:
 - a. $\forall x (p(x) \rightarrow q(x)) \rightarrow ((\forall x p(x)) \rightarrow (\forall x q(x)))$
 - b. $\forall x (p(x) \rightarrow q(x)) \rightarrow ((\exists x p(x)) \rightarrow (\exists x q(x)))$
 - c. $\exists x (p(x) \rightarrow \forall x p(x))$
 - d. $((\exists x p(x)) \rightarrow (\forall x q(x))) \rightarrow (\forall x (p(x) \rightarrow q(x)))$