1. **[10 points]** Sipser book, Exercise 1.4f:

Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts $\Sigma = \{a, b\}$.

- **f.** \{ $w | w$ has an odd number of $a$'s and ends with a $b$ \}

2. **[10 points]** Sipser book, Exercise 1.5f:

Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts $\Sigma = \{a, b\}$.

- **f.** \{ $w | w$ is any string not in $a^* \cup b^*$ \}

3. **[5 points]** Give a regular expression for the complement language above.

4. **[10 points]** Sipser book, Exercise 1.19b:

Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

- **b.** $((00)^*(11) \cup 01)^*$

5. **[10 points]** Convert the NFA in the previous problem to a DFA and minimize it.

6. **[20 points]** Sipser book, Problem 1.43b. *Also, give a non-trivial example.*

Let $A$ be any language. Define $DROP-OUT(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in $A$. Thus, $DROP-OUT(A) = \{xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma \}$. Show that the class of regular languages is closed under the DROP-OUT operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

7. **[15 points]** As in problem 6, except for MEGA-DROP-OUT:

MEGA-DROP-OUT($A$) = \{xz | \exists y \in \Sigma^* \text{ xyz } \in A \}

8. **[10 points]** Sipser book, Problem 1.45:

- **1.45** Let $A/B = \{w \mid wx \in A \text{ for some } x \in B \}$. Show that if $A$ is regular and $B$ is any language then $A/B$ is regular.

(over, please)
9. [10 points] Sipser book, Problem 1.53:

1.53 Let \( \Sigma = \{0, 1, +, =\} \) and

\[
ADD = \{ x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z \}\.

Show that \( ADD \) is not regular.