Computer Science 81, Spring 2009
Assignment 10
Due Mon. April 20
Please work each problem on a separate sheet.

1. **[15 points] Sipser book, Exercise 2.2:**

2.2 **a.** Use the languages $A = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n \mid m, n \geq 0\}$ and $B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

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2. **[10 points] Sipser book, Exercise 2.6d:**

2.6 Give context-free grammars generating the following languages.

d. $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 1, \text{ each } x_i \in \{\mathtt{a}, \mathtt{b}\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^{\mathcal{R}}\}$

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3. **[30 points]**

Give a context free grammar that generates the language $\{x \in \{0, 1\}^* \mid \#_0(x) = \#_1(x)\}$. Prove your answer. (Show that your grammar generates every such string, and also that it generates only such strings. This may require a separate proof for each direction of set inclusion.

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4. **[15 points] Sipser book, Problem 2.10,** except give a **formal** description as well.

2.10 Give an informal description of a pushdown automaton that recognizes the language A in Exercise 2.9.

$$A = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

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5. **[10 points] Sipser book, Problem 2.14:**

2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \varepsilon$$
$$B \to \mathtt{00} \mid \varepsilon$$

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6. **[20 points] Sipser book, Problem 2.33:**

2.33 Show that $F = \{\mathtt{a}^i\mathtt{b}^j \mid i \neq kj \text{ for every positive integer } k\}$ is not context free.

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7. **[Extra Credit 25 points]** Show that for any pda, there is a pda accepting the same language by the empty-stack criterion that has only *one* control state. (Be sure to cover the case where the "pushed" value in the original pda is ε.)