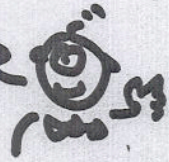


space log?

# Log Space!



- $L = \text{SPACE}(\log n)$

Here we mean  $O(\log n)$

- $NL = \text{NSPACE}(\log n)$

$\log n^{42}$  is  $O(k)$



logarithmic space?  
But the input alone has space  $n$ .



read-only input tape (2-way)



read/write work tape

Try these!

Ex 1

$$L = \{ 0^i 1^i \mid i \geq 0 \} \in L \quad ?$$
$$\in NL \quad ?$$

EX 2

PATH =  $\{ \langle G, s, t \rangle \mid G \text{ is a digraph}$   
and  $\exists$  path from  $s$  to  $t \}$

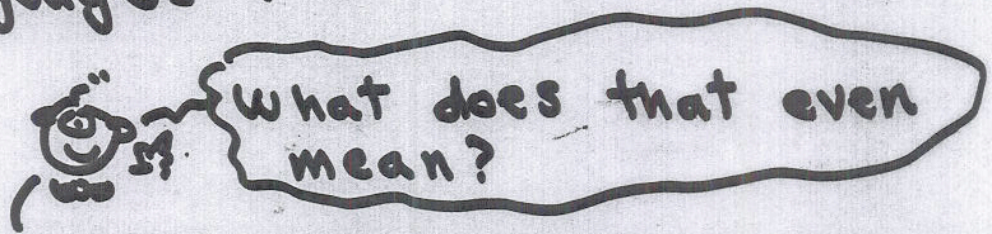
$\in L \quad ?$   
 $\in NL \quad ?$



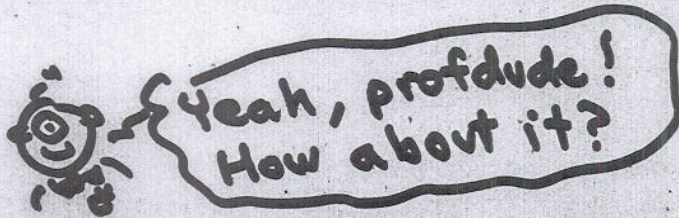
How 'bout  
DFS or BFS?

- what is the relationship between  $L$ ,  $NL$ ,  $P$ , etc?

- Are there  $NL$ -complete languages?



So, How About  
NL-completeness?



Def

A language  $\Pi$  is NL-complete  
if

1.

2.

Recall that ...

If any NP-complete language  $\pi$  is also in P then  $P = NP$

similarly, we claim that ...

If any NL-complete language  $\pi$  is also in L then  $L = NL$

A Cook/Stockmeyer Type Theorem:  
(Savitch, 1970)

We know  
it's in  
P

PATH =  $\{ \langle G, s, t \rangle \mid G \text{ is a digraph}$   
and  $\exists$  path from  $s$  to  $t \}$

is NL-complete.

That is WAY  
cool!

Finally,

Claim:  $L \subseteq NL \subseteq P$

$L \subseteq NL \subseteq P$

$= co-NL$



