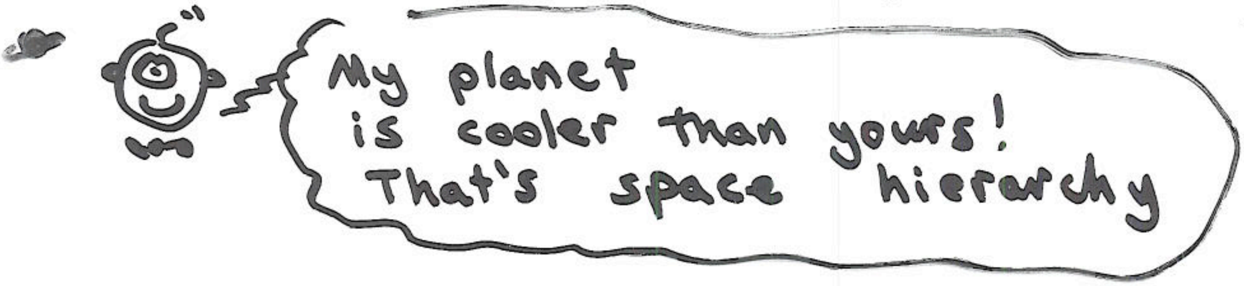
Space Hierarchy Theorem



Theorem: For any <u>space constructible</u> function f $\exists$ a language L decidable in $O(f(n))$ space s.t. L is NOT decidable in $o(f(n))$ space.

"little o"

$g(n) \in o(f(n))$ if $\forall c > 0$
$\exists n_0$ s.t. $c \cdot g(n) < f(n)$
$\forall n \geq n_0$

$n^2 \in o(n^3)$

$n^3 \in o(42n^3)$?

Def: A function $f(n)$ is space constructible if … $\exists$ a TM M s.t. …
… on any input w of length n
M marks of position $f(n)$ on tape
& never trespasses!

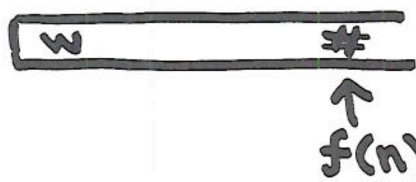
Even more space for rent!

in DSPACE (f(n))

Description of L by constucting a TM M_L :

$|w| = n$

1. on input w, space construct f(n)



2. Check if input w is of form $\langle M \rangle 0^*$ If not REJECT !

3. Simulate M on $w = \langle M \rangle 0^*$
   REJECT if we trespass the #

4. Use f(n) space to keep a counter with max value $2^{f(n)}$
   REJECT if we timeout

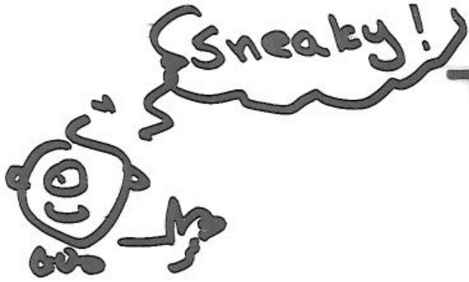
   increment counter on each simulation step of M on w

whew!

5. If M halts and accepts w we REJECT

   Diagonalization!

   If M halts and rejects w we ACCEPT



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Borodin's Gap Theorem

Diagonalization on steroids!

Theorem: $\exists$ $f(n)$ s.t. any language decided in time $2^{f(n)}$ is also decided in time $f(n)$

No big-Oh here!
Wow! There's a huge gap between $f(n)$ and $2^{f(n)}$!

Hey!!
Doesn't this contradict the time hierarchy theorem?!

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a gap in content