What is this?

- A type of computation model and declarative programming language.
- It accommodates a different mind-set from conventional languages.
- Uses:
  - Artificial intelligence applications
  - Language understanding and translation
  - Knowledge representation
  - Databases

Why study this?

- Expands expressiveness over what we have seen so far.
- A good language for learning about computational paradigms based on:
  - Certain forms of logic
  - Non-determinism
  - Backtracking

Who uses Prolog?

- People who want to have a broad set of intellectual and problem-solving tools at their disposal.
- People who develop systems based on knowledge and reasoning.
- People who are not afraid of the unconventional.

Advantages of Logic-Programming Succinctness

- Code moves closer to concepts (and farther from machine details).
- Multiple purposes served by a single piece of code.
- Easier to ascertain correctness.
- Easier to modify, for purposes of software evolution.

Prolog’s Origins

(see [http://en.wikipedia.org/wiki/Prolog](http://en.wikipedia.org/wiki/Prolog))

Prolog evolved out of research on language translation at the University of Aix-Marseille back in the late 60’s and early 70’s.

Alain Colmerauer and Philippe Roussel, both of University of Aix-Marseille, collaborated with Robert Kowalski of the University of Edinburgh to create the underlying design of Prolog as we know it today.

Kowalski contributed the theoretical framework on which Prolog is founded, while Colmerauer’s research at that time provided means to formalize the Prolog language.

Colmerauer

Kowalski
The ISO Prolog standard: ISO/IEC 13211-1 published in 1995, aims to standardize the existing practices of the many implementations of the core elements of Prolog. It has clarified aspects of the language that were previously ambiguous and leads to portable programs. The standard is maintained by the ISO X3J17 committee.

ISO = "International Standards Organization"

Logic vs. Functions

- Logic programming and functional programming are two types of declarative programming.
- In functional programming, a high-level solution function is realized by composing simpler functions.
- In logic programming, a predicate expressing a goal that the solution is expected to fulfill is decomposed into simpler predicates, using logic.

Predicate vs. Function Preview

- Function append:
  
  \[
  (\text{append } '(a \ b \ c) '(d \ e)) \Rightarrow '(a \ b \ c \ d \ e)
  \]

- Predicate append:

  \[
  \text{append}([a, b, c], [d, e], Z). \Rightarrow Z = [a, b, c, d, e]
  \]

But Predicates Offer More Options

- Predicate append:
  
  \[
  \text{append}([a, b, c], [d, e], Z). \\
  \text{append}([a, b, c], Y, [a, b, c, d, e]). \\
  \text{append}(X, [a, b, c, d, e]). \\
  \Rightarrow X = [], Y = [a, b, c, d, e]. \\
  \Rightarrow X = [a], Y = [b, c, d, e]. \\
  \Rightarrow X = [a, b], Y = [c, d, e].
  \]

- Tying these ideas together into a coherent logical framework is the contribution of logic programming.

From the beginning... Varieties of Logic

- Proposition Logic:
  
  - Propositions are symbols which may be assigned one of two values:
    - true
    - false
  
  - without regard to specific individuals.

- Predicate Logic:
  
  - Predicates can be viewed functions from a domain of individuals to \{(true, false)\}.
  
  - The domains are, e.g. strings, numbers, lists, and various other structures.

Comparison

- Propositions:

  - \text{hmc\_is\_great}
  - \text{caltech\_is\_in\_glendale}

- Predicates (X and Y are variables)

  \[
  \text{domain} = \{\text{caltech, glendale, hmc, ...}\}
  \]

  - \text{is\_great}(X) \text{ is\_great(hmc)}
  - \text{is\_in}(X, Y) \text{ is\_in(caltech, glendale)}
Logic of Implication (Prolog Style)

- Logical rule:
  - $Q :\neg P1, P2, P3$,
  - means that proposition or predicate application $Q$ is implied by the conjunction of $P1, P2,$ and $P3$.
  - If each of $P1, P2, P3$ are true, then $Q$ is true.
  - If one of $P1, P2, P3$ is false, nothing is claimed.

Prolog Lingo: Clauses

- This is a clause:
  - $Q :\neg P1, P2, P3$,
  - $Q$ is called the head,
  - $P1, P2, P3$ comprise the body,
  - Each of $Q, P1, P2, P3$ are individually called goals.

Example

- Things to do to prepare for an exam.
  - Attend all the lectures.
  - Read the book.
  - Do the problems.
  - Be tutored by someone.
  - or, maybe you already know it all.

Example Clause

- prepared_for_exam :\neg
  - read_book,
  - worked_problems,
  - attended_lectures.

Example Clause

- prepared_for_exam :\neg
  - knows_it_all.

Example Clause

- prepared_for_exam :\neg
  - tutored_by_someone_prepared.
**Facts**

- Facts are clauses with an empty body.
- They assert the truth of something without qualification.

**Examples of Possible Facts**

- read_book.
- worked_problems.
- attended_lectures.
- knows_it_all.
- tutored_by_someone_prepared.

Depending on the facts present, a goal prepared_for_exam maybe inferred as true or not.

**Success and Failure**

- A goal is presented interactively to Prolog as:
  
  \[\text{?- prepared_for_exam.}\]

  Depending on the facts, this goal may succeed or fail.

**Success and Failure Examples**

- If the facts are:
  
  attended_lectures.
  read_book.
  worked_problems.

  and there is just the clause:

  \[\text{prepared_for_exam :- read_book, worked_problems, attended_lectures.}\]

  then the goal

  \[\text{?- prepared_for_exam.}\]

  succeeds. If any of the facts is missing, the goal fails.

**Success and Failure**

- A goal succeeds provided one of:
  
  - There is a fact that matches the goal, or
  - There is a clause, the head of which matches the goal, and all goals in the body succeed.

  [Notice the first blue bullet is really a special case of the second.]

- If a goal doesn’t succeed, then it fails.

**Success and Failure Examples**

- If the facts are:
  
  read_book.
  attended_lectures.
  tutored_by_someone_prepared.

  and there is a clause:

  \[\text{prepared_for_exam :- tutored_by_someone_prepared.}\]

  then the goal

  \[\text{?- prepared_for_exam.}\]

  succeeds.
Two Compatible Interpretations of Prolog Execution

- **Logical interpretation:**
  - Implications and facts, logical proof.
  - `prepared :- read, worked, attended.`
  - "If you've read, worked, and attended, then you're prepared."

- **Procedural interpretation:**
  - Goals, backtracking, etc.
  - "To prepare, (you can) read, work, attend."

How Prolog Works: Depth-First Search

- In general, there can be several goals, all of which need to succeed.
- Think of these goals as being kept in a stack.
- **Success** occurs when the stack is empty.
- The first goal is removed from the stack.
- Prolog searches for a clause having a matching head.
  - If none is found, then there is overall failure.
  - If a matching fact is found, then execution continues with the rest of the stack.
  - If a matching clause is found, then the clauses in the body of the clause are pushed onto the stack (with the leftmost goal now at the top) and execution continues with the new list.

Backtracking

- When a failure occurs, Prolog does not necessarily stop. It goes back to the previous clause it used, and restores the list to the way it was just before. (It "backtracks").
- It then tries any alternative clauses that follow that clause.
- Clauses are tried in the order listed in the program, until there are no more clauses left.
- For each new goal on the stack, searching starts afresh.

Backtracking Example

- Suppose the clauses and facts are:
  - `prepared_for_exam :-
    read_book,
    worked_problems,
    attended_lectures.`
  - `prepared_for_exam :-
    tutored_by_someone_prepared.`
  - `read_book.`
  - `worked_problems.`
  - `tutored_by_someone_prepared.`
- Consider the goal `?- prepared_for_exam.`
- The initial goal stack is:
  - `[prepared_for_exam]`
- After a match with the first clause, the stack becomes:
  - `[read_book, worked_problems, attended_lectures]`
- Then, as facts are matched away, the stack becomes:
  - `[attended_lectures]`
- But here there is no matching fact or clause. Backtracking occurs, eventually restoring the list to:
  - `[prepared_for_exam]`
- The next clause that matches changes the list to:
  - `[tutorial_by_someone_prepared]`
- The first goal matches a fact, leaving:
  - `[]`
- The original goal thus succeeds.

Prolog's Version of Negation

- Facts can only be asserted in the positive sense.
- Negation can be tested, but not asserted.
- Negation is "negation as failure": `\+Goal` succeeds iff `Goal` fails.
- Etymology: In classical logic, |− G means "G is provable". \+ is a "smiley" version of "is not provable".
A Negation Example

The clause
prepared_for_exam :-
read_book,
worked_problems,
attended_lectures,
\+ slept_during_lectures.

will enable prepared_for_exam to succeed by this clause only if slept_during_lectures does not succeed.

Predicate vs. Propositional Goals

The goals so far have been propositional:
- Each is either invariably true (succeeds) or false (fails).
- Using predicates, success or failure depends on arguments.
- Each fact and clause can have one or more arguments.

Exam Passing for the Full Class

Let's say the class contains \{bob, fred, judy, sam\}, and the facts are:
- read_book\( (fred) \).
- read_book\( (judy) \).
- worked_problems\( (judy) \).
- attended_lectures\( (fred) \).
- attended_lectures\( (judy) \).
- tutored_by\( (fred, bob) \).
- tutored_by\( (sam, judy) \).

Predicate Form of Exam Passing

prepared_for_exam\( (X) \) :-
read_book\( (X) \),
worked_problems\( (X) \),
attended_lectures\( (X) \).

prepared_for_exam\( (X) \) :-
tutored_by\( (X, Y) \),
predpared_for_exam\( (Y) \).

Extreme Case-Sensitivity

In Prolog:
- Variables always start with upper-case or underscore `_`.
- Things that start with lower-case are:
  - Predicates, propositions
  - Data items (atoms): Similar to symbols in Racket/Scheme
  - Data items can also start with upper-case if singly-quoted, e.g. ‘John Hancock’.
  - Unlike Racket/Scheme, dash `-` is not considered to be just another letter. (It is an infix “functor”)

Case Sensitivity, Arity

- read_book\( (fred) \).
- prepared_for_exam\( (X) \) :-
  tutored_by\( (X, Y) \),
predpared_for_exam\( (Y) \).

Variables:
- \( X, Y \)

Atoms:
- \( fred \)

Predicates:
- read_book/1, prepared_for_exam/1, tutored_by/2

\( N \) indicates the arity (number of arguments) of the predicate. Predicate names can be overloaded.
Matching = Unification

- **Unify** means: "make the same".
- Two atoms are unifiable iff identical.
- A variable can be unified with anything (even another variable), by substituting the latter thing for the variable.
- Two predicate expressions can be unified, provided that:
  - The predicate names are identical.
  - The number of arguments is the same in both.
  - Each of the arguments can be pairwise-unified, by a common substitution.

### Unification Examples

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Unifiable?</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>fred</td>
<td>bob</td>
<td>No</td>
<td>X ← fred</td>
</tr>
<tr>
<td>fred</td>
<td>X</td>
<td>Yes</td>
<td>X ← fred</td>
</tr>
<tr>
<td>X</td>
<td>bob</td>
<td>Yes</td>
<td>X ← bob</td>
</tr>
<tr>
<td>X</td>
<td>Y</td>
<td>Yes</td>
<td>X ← Y</td>
</tr>
<tr>
<td>p(X, Y)</td>
<td>p(fred, bob)</td>
<td>Yes</td>
<td>X ← fred</td>
</tr>
<tr>
<td>p(X, bob)</td>
<td>p(fred, Y)</td>
<td>Yes</td>
<td>X ← fred</td>
</tr>
<tr>
<td>p(X, bob)</td>
<td>p(fred, X)</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

### More Unification Examples

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Unifiable?</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(X, X)</td>
<td>p(fred, bob)</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>p(X, f(Y))</td>
<td>p(Y, Z)</td>
<td>Yes</td>
<td>X ← Y, Z ← f(Y), Y ← X, Z ← f(X)</td>
</tr>
<tr>
<td>p(a, f(Y))</td>
<td>p(Y, Z)</td>
<td>Yes</td>
<td>Y ← a, Z ← f(a)</td>
</tr>
<tr>
<td>p(g(Z), f(Y))</td>
<td>p(Y, Z)</td>
<td>Yes (but only in Prolog)</td>
<td>Y ← g(f(g(*))), Z ← f(g(f(*)))</td>
</tr>
</tbody>
</table>

### Quiz 1: Complete the Table

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Unifiable?</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(X, fred)</td>
<td>p(fred, X)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(X, Y)</td>
<td>p(Y, Z)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(b, f(b))</td>
<td>p(f(b), b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(a, X)</td>
<td>p(a, f(a))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(X, f(Z), Z)</td>
<td>p(a, X, a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(X, Y, Z)</td>
<td>p(f(Y), g(Z), a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Use = in Command to Check Unifiability

% This is with SWI-Prolog 10.5.2

?- p(a, f(Y)) = p(Y, Z).
    Y = a,
    Z = f(a).

?- p(X, X) = p(fred, bob).
    false.

?- p(g(Z), f(Y)) = p(Y, Z).
    Z = f(g(**)),
    Y = g(f(**)).

### Unification and Clause Searching

- The variables of a clause are purely local to the clause. Variables do not connect one clause to another.
- When searching for a match, the variables in a clause are first renamed uniformly across the clause so that they are distinct from any other variables that might be in the goal.
- Any substitution applied to a goal is applied to all remaining goals in the list.
Example

Clauses:
- prepared_for_exam(bob).
- tutored_by(fred, bob).
- tutored_by(X, Y).
- prepared_for_exam(Y).

Goals:
- [prepared_for_exam(fred)]

Substitution:
- X1 ← fred

New goals:
- [tutored_by(fred, Y1), prepared_for_exam(Y1)]

Substitution:
- Y1 ← bob

New goals:
- [prepared_for_exam(bob)]

Recursive Example

Clauses:
- child(Ancestor, Child, Graph) :- member([Ancestor, Child], Graph).
- isDescendant(Ancestor, Desc, Graph) :- child(Desc, Child, Graph).
- isDescendant(Ancestor, Desc, Graph) :- child(Ancestor, Child, Graph), isDescendant(Child, Desc, Graph).

?- isDescendant(e, a, [[a,b], [b, c], [c,d], [b,e], [a,f]]).

Yes

Prolog’s Data Types

- Atoms: x, abc, y99, this_is-too
- Numbers: 789, 15.3e-27
- Terms: f(x, 789)
- Lists (special type of term):
  - [red, green, blue]
  - [[red, 10], [green, 20], [blue, 50]]

Throw-Away Variable

Variables beginning with _ (including _ itself) prevent the Prolog compiler from complaining about "singleton variables" in clauses (which usually signals a user error).

p(X1) :- q(X2). % Did the user intend X1 and X2 to be the same?
p(_X1) :- q(_X2). % If so, do it this way

Warning: ... filename ... line containing first clause ...
Singleton variables: [X1,X2]

Throw-Away Examples

?- _X = 1, _X = 2. % Each _X is the same var
false.

?- _ = 1, _ = 2. % Each _ is a separate var
true.
Database Applications

- Data are stored as predicate facts (aka "relations").
- Queries are goals.
- Substitutions (resulting from unifications) are results.

Relational Database Example

<table>
<thead>
<tr>
<th>Name</th>
<th>Dorm</th>
<th>Dept</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>East</td>
<td>CS</td>
<td>60</td>
</tr>
<tr>
<td>Naima</td>
<td>South</td>
<td>CS</td>
<td>80</td>
</tr>
<tr>
<td>Alice</td>
<td>West</td>
<td>CS</td>
<td>5</td>
</tr>
<tr>
<td>Roy</td>
<td>North</td>
<td>Math</td>
<td>30</td>
</tr>
<tr>
<td>Albert</td>
<td>South</td>
<td>Math</td>
<td>70</td>
</tr>
<tr>
<td>Toshiko</td>
<td>East</td>
<td>Math</td>
<td>50</td>
</tr>
</tbody>
</table>

Three relations:
- `lives ⊆ names x dorms` (as a set of pairs)
- `takes ⊆ names x depts x numbers` (as a set of 3-tuples)
- `tutors ⊆ names x depts x numbers`

Relational Database Example

Sample Queries:
- Who lives in South dorm?
  \[
lives(X, 'South')
  \]
- Who lives in East dorm and takes CS 60?
  \[
lives(X, 'East'), takes(X, 'CS', 60)
  \]
- Who takes a CS course?
  \[
takes(X, 'CS', _)
  \]

Quiz 2

Express as Prolog Queries:
- Who takes a CS course and tutors a Math course?
  \[
canTutor(X, Y) :-
tutors(X, Dept, Number),
takes(Y, Dept, Number).
  \]
- What tutors live in West dorm?
  \[
lives(john, east).
lives(naima, south).
lives(alice, west).
lives(toshiko, east).
lives(roy, north).
lives(albert, south).
  \]
- Who lives in East dorm that is not a tutor?
  \[
  \]
- Who lives in East dorm that is not a tutor?
  \[
  \]
  Previous Example Prolog KB

Solving Goals with Variables

- Variables get bound during matching.
- They get unbound during backtracking, but never before.
- After backtracking, they may be re-bound.
- Amazingly, this all can be viewed declaratively.
Goal Succession:
Depth-First Execution in Prolog: Query 1

canTutor(alice, Y).

variable, since starts with upper-case

Goal Succession:
Depth-First Execution in Prolog: Chaining
canTutor(alice, Y).

tutors(alice, Dept, Number), takes(Y, Dept, Number).

Goal Succession:
Depth-First Execution in Prolog: Binding
canTutor(alice, Y).
tutors(alice, Dept, Number), takes(Y, Dept, Number).
tutors(alice, math, 55).
takes(Y, math, 55)

Goal Succession:
Depth-First Execution in Prolog: Result 1a
canTutor(alice, Y).
tutors(alice, Dept, Number), takes(Y, Dept, Number).
tutors(alice, math, 55).
takes(Y, math, 55).
takes(roy, math, 55).

Y = roy

Goal Succession:
Undoing Binding on Failure
canTutor(alice, Y).
tutors(alice, Dept, Number), takes(Y, Dept, Number).
tutors(alice, math, 55).
takes(Y, math, 55).
takes(roy, math, 55).

Goal Succession:
Retrying
canTutor(alice, Y).
tutors(alice, Dept, Number), takes(Y, Dept, Number).
tutors(alice, math, 55).
takes(Y, math, 55).
Backtracking in Depth-First Search
Rebinding: Result 1b

Deeper Backtracking: Query 2, Result 2a

Summary of Backtracking

- Given a goal, Prolog tries rules in order of occurrence ("top-to-bottom"), using the first rule, the consequent of which matches the goal.
- If the rule has sub-goals, the sub-goals are satisfied in order of occurrence ("left-to-right"), resulting in bindings at each stage.
- If a goal sub-goal fails completely, Prolog retries to satisfy it using the next available option (e.g. the next rule).
Rule and Sub-Goal Ordering

Suppose the goal is `knows(john, Y, R).

This rule is tried first.

This rule is tried after the first rule is exhausted.

This sub-goal is satisfied first, binding Z.

This sub-goal is satisfied next.

In effect, we have disjunction (or) among rules, and conjunction (and) within rules. Remember that Prolog execution is depth-first search.

And-Or Trees

- In AI, problem-solving trees are typically "And-Or" trees.
- This applies to Prolog's goals.

\[ \text{conjunction ("and") of goals (within rule)} \]
\[ \text{disjunction ("or") of goals (among rules)} \]
\[ \text{arrow denotes possible solution} \]

"Logical Variables" in Prolog

- Ideally, variables are understood as pure logic. However, a deeper operational understanding is sometimes necessary.
- A variable in Prolog is like an object that can have one of two states:
  - unbound
  - bound, to some Prolog term, e.g. an individual
- Once the variable is bound, it only gets re-bound in backtracking, which results in first unbinding the previous binding.

Creation of Logical Variables

- Logical variables may get created dynamically and implicitly whenever goals are introduced.
- A top-level goal may contain logical variables.
- The use of a clause, the head of which matches a goal, may create new goals containing logical variables.

Logical Variable Binding is Transitive

- One logical variable can be bound to another, which means that they are effectively the same, as long as this binding is in effect.
- Arbitrarily-long chains of bindings can exist.

Example

- \( X = b, Y = X. \)

  \( X \) is first bound to \( b \), then \( Y \) is bound to \( X \).

  Both \( X \) and \( Y \) effectively have the same value, \( b \).
Example

- $X = Y$, $Y = b$.

  $X$ is first bound to $Y$, then $Y$ is bound to $b$.

  Both $X$ and $Y$ effectively have the same value, $b$.

Example continued

?- p(Z).

Z is a logical variable

$Z = b$ will be the first solution.

On backtracking, $Z$ will be re-bound to $X$ in the second clause. A new logical variable for $Y$ is introduced.

$Z$ and $Y$ will then have values $c$, $b$.

$Z = c$ will be the second solution.

Lists in Prolog

- Suppose we have some clauses for predicates $p$ and $q$:
  - $p(b)$.
  - $p(X) \leftarrow q(X, Y), p(Y)$.
  - $q(c, b)$.

  What solutions will be produced for the top-level goal:
  
  ?- p(Z).

Logical Variables in Lists and Other Structures

- Because they are "objects", variables can occur in lists in either bound or unbound states.

  Suppose $L = [a, X, b]$.

  If $X$ is unbound, it acts as a "place-holder", which can subsequently be bound by unification, e.g.

  $L = [\_, c, \_]$ will unify $X$ with $C$.
Movie Database

- movie([Title, Year], Director, Categories), e.g.
  movie(["Being John Malkovich", 1999], "Spike Jonze").
- actress(Name, [Birth City, State], Year), e.g.
- actor(Name, [Birth City, State], Year), e.g.
- play([Player, Part, [Title, Year]], e.g.
  play(["Ben Affleck", "Rafe", ["Pearl Harbor", 2001]])

Numeric Aspects

- Numbers can be compared like any other goal:
  - 2 < 3 succeeds
  - 5 <= -5 fails
- Numeric comparisons (caution):
  - <, >, =<, =>, =\=, =:=

Numeric Operators: Different!!!

- 2+3 is not 5
  It is an unevaluated term, effectively +(2, 3).
- The 'is' operator causes evaluation:
  X is 2+3
  binds X to 5.

Functional Programming in Prolog

- Most concepts you know from Racket/Scheme are applicable.
- Syntax is different.
- Also, "higher order" functions are not built-in. We must code them.

Example: range function

- range(M, N, []) :- M > N.
- range(M, N, [M | L]) :-
  M is N,
  M1 is M+1,
  range(M1, N, L).

?- range(1, 10, L).
L = [1,2,3,4,5,6,7,8,9,10]

Note: This range is not "reversible", due to the use of is.
Example: max computes the maximum of a non-empty list of numbers

```
max([X | L], M) :- max_helper(L, X, M).
max_helper([], M, M).
max_helper([X | L], M, N) :- X > M, max_helper(L, X, N).
max_helper([X | L], M, N) :- X =< M, max_helper(L, M, N).
```

test :- max([3, 7, 2, 9, 1, -5], M), write(M), nl.

Note: This max is not "reversible", due to the use of >.

Note: Tail recursion applies here.

Using McCarthy's Transformation

- In some cases, either think, or write out, the program as imperative, then convert to logic.

Example: extended max computes the maximum of a list of numbers and the first location of that maximum

```
max([X | L], M) :- max_helper(L, X, M).
max_helper([], M, M).
max_helper([X | L], M, N) :- X > M, max_helper(L, X, N).
max_helper([X | L], M, N) :- X =< M, max_helper(L, M, N).
```

test :- max([3, 7, 2, 9, 1, -5], M), write(M), nl.

Note: This max is not "reversible", due to the use of >.

Note: Tail recursion applies here.

Example: extended max computes the maximum of a list of numbers and the first location of that maximum

Imperative Version:

```
L is the original list
M will be the maximum value
I will be the first location of M
L = ... non-empty list ...
N = first(L);
K = 0; // location of N
L = rest(L);
J = 1; // location of first element of L
while( L is non-empty ) {
    if( first(L) > N ) {
        N = first(L); K = J;}
    J = J+1;
    L = rest(L);
}
M = N;
I = K;
```

Logic Version:

```
max([X | L], M, I) :- max_helper(L, X, M, 0, M, I).
max_helper([], 1, X, 0, X, 0, I).
max_helper([X | L], J, N, _K, M, I) :- X > N, J1 is J+1, max_helper(L, J1, X, J, M, I).
max_helper([X | L], J, N, K, M, I) :- X =< N, J1 is J+1, max_helper(L, J1, N, K, M, I).
```

Using -> ... ; ... (if ... then ... else ...) for greater clarity and efficiency

Original Version:

```
max([X | L], M, I) :- max_helper(L, X, 0, M, I).
max_helper([], 1, X, 0, M, I).
max_helper([X | L], J, N, _K, M, I) :- X > N, J1 is J+1, max_helper(L, J1, X, J, M, I).
max_helper([X | L], J, N, K, M, I) :- X =< N, J1 is J+1, max_helper(L, J1, N, K, M, I).
```

Revised Version:

```
max([X | L], M, I) :- max_helper(L, X, 0, M, I).
max_helper([], 1, X, 0, M, I).
max_helper([X | L], J, N, _K, M, I) :- J1 is J+1, % Note outer (...) needed for correctness.
                          X > N =>
                          max_helper(L, J1, X, J, M, I);
                          max_helper(L, J1, X, J, M, I).
max_helper([X | L], J, N, K, M, I) :- J1 is J+1, max_helper(L, J1, N, K, M, I).
```
call: Predicates as Arguments

- \text{call}(P, A_1, A_2, \ldots, A_n) \text{ allows predicate } P \text{ to be a variable.}
- Its meaning is as if \( P(A_1, A_2, \ldots, A_n) \)
- The latter syntax is not allowed, however.

call example

\begin{verbatim}
p(0, 1).
p(1, 2).
p(2, 0).

test(X) :- call(p, X, Y), write(Y), nl.

?- test(0).
true.

?- test(1).
true.

?- test(2).
true.
\end{verbatim}

map example:
map applied to 2-ary predicates

\begin{verbatim}
map(_, [], []).
map(P, [A | X], [B | Y]) :-
    call(P, A, B),
    map(P, X, Y).

% Example use
p(X, Y) :- Y is X+1.
test :- map(p, [1, 2, 3], Z), write(Z), nl.
\end{verbatim}

More on Using Logical Variables

- length(X, N) \text{ is true when } X \text{ is a list of length } N.
- length([a, b, c], 3) succeeds
- length([a, b, d], 3) fails
- length(X, 3) succeeds with \( X \) being a list of three generated logical variables

\begin{verbatim}
?- length(X, 3).
X = [c | G263, _G266, _G269].
\end{verbatim}

More on Using Logical Variables

- member(A, L) \text{ succeeds when } A \text{ is a member of list } L.
- member(c, [a, b, c]) succeeds.
- member(c, [a, b, d]) fails.
- member(c, [a, b, X]) succeeds with \( X = c \).
- member(c, X) succeeds an infinite number of ways.

\begin{verbatim}
?- member(c, X).
\end{verbatim}

Generate-Test Programming

- One way to solve a constraint-based problem:
- Generate a trial solution
- Test to see if it really is a solution
- Repeat the above until a solution is found.
- A challenge is to get the generation to cover all possibilities.
Generate-Test Programming

Consider:

\[ \text{member}(X, [X | _]). \]
\[ \text{member}(X, [_ | L]) \rightarrow \text{member}(X, L). \]

This predicate can be viewed as a member tester. It can also be viewed as a member generator.

Generating vs. Testing

**test**

?- \text{member}(3, [1, 2, 3, 4, 5]).
yes

?- \text{member}(6, [1, 2, 3, 4, 5]).
no

**generate**

?- \text{member}(X, [1, 2, 3, 4, 5]).

\[ \text{X} = 1 ; \]
\[ \text{X} = 2 ; \]
\[ \text{X} = 3 ; \]
\[ \text{X} = 4 ; \]
\[ \text{X} = 5 ; \]
no

A simple constraint problem

- I want a list \( L \) that:
  - Contains exactly three elements
  - All elements are members of a list \( M \)
  - One element occurs exactly twice in the list

- Simple solution:
  - \text{solve}(\{X, X, Y\}, M) \rightarrow \text{ok}(X, Y, M).
  - \text{solve}(\{X, Y, X\}, M) \rightarrow \text{ok}(X, Y, M).
  - \text{solve}(\{Y, X, X\}, M) \rightarrow \text{ok}(X, Y, M).
  - \text{ok}(X, Y, M) \rightarrow \text{member}(X, M), \text{member}(Y, M), X \neq Y.

Example Execution

?- \text{solve}(L, [1, 2, 3]).

\[ L = \{1, 1, 2\} ; \]
\[ L = \{1, 1, 3\} ; \]
\[ L = \{2, 2, 1\} ; \]
\[ L = \{2, 2, 3\} ; \]
\[ L = \{3, 3, 1\} ; \]
\[ L = \{3, 3, 2\} ; \]
\[ L = \{2, 2, 3\} ; \]
\[ L = \{2, 2, 3\} ; \]
\[ L = \{2, 3, 2\} ; \]
\[ L = \{3, 1, 2\} ; \]
\[ L = \{3, 2, 1\} ; \]
\[ L = \{3, 1, 3\} ; \]
\[ L = \{3, 1, 3\} ; \]
\[ \text{false}. \]

What Happens

?- \text{solve}(L, [1, 2, 3]).
Clause \text{solve}(\{X, Y, Y\}, M) \rightarrow \text{ok}(X, Y, M).
binds \( L \) to \( \{X, Y, Y\} \), creating goal

\text{ok}(X, Y, \{1, 2, 3\})
which is solved to get \( X = 1, Y = 2 \)
which gives \( L = \{1, 1, 2\} \)
backtracking gives \( X = 1, Y = 3 \)
which gives \( L = \{1, 1, 3\} \)
backtracking gives \( X = 2, Y = 1 \)
which gives \( L = \{2, 2, 1\} \)
backtracking gives \( X = 2, Y = 3 \)
which gives \( L = \{2, 2, 3\} \)

etc: eventually other clauses for solve are used.

Generate/Test Example: Map Coloring

A map

A map coloring problem is shown, with nodes A, B, C, D, E, F, and G connected in a graph.
**Map Coloring (2)**

A map

```
A   B
C   D
E   F
G
```

Corresponding graph

```
A   B
C   D
E   F
G
```

**Map Coloring (3)**

Prolog Clause

```
map([A, B, C, D, E, F, G]) :-
  next(A, B),
  next(A, C),
  next(A, D),
  next(A, E),
  next(B, D),
  next(B, F),
  next(C, D),
  next(C, E),
  next(C, F),
  next(D, F),
  next(E, F),
  next(E, G),
  next(F, G).
```

Graph

```
A   B
C   D
E   F
G
```

**Map Coloring (4): Color Constraints**

```
next(X, Y) :- color(X), color(Y), X != Y.

color(red).
color(blue).
...
```

These and the preceding clause are the entire program.

**Version where the colors are presented as a list**

```
next(X, Y, Colors) :-
  member(X, Colors, Residue),
  member(Y, Residue).
```

The generalized member ensures that the second color is not a duplicate (assuming no duplicates in the original list).

**Version where the colors are presented as a list**

```
map([A,B,C,D,E,F,G], Colors) :-
  next(A, B, Colors),
  next(A, C, Colors),
  ...
```

**Sudoku**

- Sudoku is basically a graph-coloring problem, except that we have a "hypergraph" rather than an undirected graph.
- Adjacency is no longer binary. Any squares in the same row, column, or sub-square are considered "adjacent".
Sudograph (pseudo-graph)

- $G =$
  - List of Nodes (logical variables)
  - List of constraints
    - Each constraint is a list of variables
    - No two variables in the same constraint can have the same node values.

Sudograph setup

```prolog
testSudograph(Nodes, Constraints, Colors):-
  Nodes = [A, B, C, D, E, F, G],
  Constraints = [[A, B, C], [B, C, D], [C, D, E], [D, E, F], [E, F, G], [G, A, B]],
  Colors = [red, blue, green, yellow],
  sudographSolver(Nodes, Constraints, Colors).

?- testSudograph(Nodes, Constraints, Colors).

Colors: [red, blue, green, yellow]
Nodes: [red, blue, green, red, blue, green, yellow]
Constraints: [red, blue, green]
[blue, green, red]
[green, red, blue]
[red, blue, green]
[yellow, red, blue]
```

The “Zebra” Problem
(aka “Einstein’s Riddle?”)

Five people of different nationalities, with different occupations, live in consecutive houses on a street. These houses are painted different colors. Each person has a different pet and a different favorite drink.

Given:
1. The English person lives in the red house.
2. The Spanish person owns a dog.
3. The green house is on the right side of the white house.
4. The Italian drinks tea.
5. The Norwegian lives in the first house on the left.
6. The photographer breeds snails.
7. The Norwegian’s house is next to the blue one.
8. The Japanese person is a painter.
9. The fox is in a house next to that of the physician.
10. The diplomat lives in the yellow house.
11. The owner of the green house drinks coffee.
12. The violinist drinks orange juice.
13. The horse is in a house next to that of the diplomat.
14. Milk is drunk in the middle house.

Determine: who owns the zebra?; who drinks water?

Analysis

- There are 5 "houses", which can be represented as a parenthesized structure:
  - (Nationality, Colo, Occupation, Pet, Drink)
- There is a list of 5 houses:
  - L = [_, _, _, _, _]
- Note that order is important in the list (left-to-right).
- Each clue places a constraint on the list.
- The questions to be answered are:
  - Find X where member((X, _, _, zebra, _), L).
  - Find Y where member((Y, _, _, _, water), L).

Translating Clues

1. The English person lives in the red house.
   ```prolog
   clue1(L) :- nationality(english, H), color(red, H), house(H, L).
   ``
2. The Spanish person owns a dog.
   ```prolog
   clue2(L) :- nationality(spanish, H), pet(dog, H), house(H, L).
   ``
3. The green house is on the right side of the white house.
   ```prolog
   clue3(L) :- color(green, G), color(white, W), rightof(G, W, L).
   ``

Helpers:
- house(X, L) :- member(X, L).
- rightof(X, Y, L) :- rightof(X, Y, L).
- leftof(X, Y, L) :- leftof(X, Y, L).
```

Unifying with House Structures

```prolog
nationality(N, (N, _, _, _, _)).
color(C, (_, C, _, _, _)).
occupation(O, (_, _, O, _, _)).
pet(P, (_, _, _, P)).
drink(D, (_, _, _, _, D)).
```
Using the Clues Together:

```prolog
clue(L) :-
  clue1(L),
  clue2(L),
  clue3(L),
  clue4(L),
  clue5(L),
  clue6(L),
  clue7(L),
  clue8(L),
  clue9(L),
  clue10(L),
  clue11(L),
  clue12(L),
  clue13(L),
  true.
```

```prolog
solution(Z, W, L) :-
  clue(L),
  pet(zebra, H1),
  nationality(Z, H1),
  house(H1, L),
  drink(water, H2),
  nationality(W, H2),
  house(H2, L).
```

Quiz 3: Translate the rest of the clues.

1. The English person lives in the red house.
   ```prolog
   clue1(L) :- nationality(english, H), color(red, H), house(H, L).
   ```

2. The Spanish person owns a dog.
   ```prolog
   clue2(L) :- nationality(spanish, H), pet(dog, H), house(H, L).
   ```

3. The green house is on the right side of the white house.
   ```prolog
   clue3(L) :- color(green, G), color(white, W), rightof(G, W, L).
   ```

4. The Italian drinks tea.

5. The Norwegian lives in the first house on the left.
   ```prolog
   clue5(L) :- nationality(norwegian, H), house(H, L), location(leftmost, H).
   ```

6. The photographer breeds snails.
   ```prolog
   clue6(L) :- pet(snail, H), nationality(photographer, H), house(H, L).
   ```

7. The Norwegian's house is next to the blue one.
   ```prolog
   clue7(L) :- nationality(norwegian, H), color(blue, H2), neighbor(H, H2).
   ```

8. The Japanese person is a painter.
   ```prolog
   clue8(L) :- nationality(japanese, H), occupation(painter, H), house(H, L).
   ```

9. The fox is in a house next to that of the physician.
   ```prolog
   clue9(L) :- pet(fox, H), nationality(physician, H2), neighbor(H, H2).
   ```

10. The diplomat lives in the yellow house.
    ```prolog
        clue10(L) :- nationality(diplomat, H), color(yellow, H), house(H, L).
    ```

11. The owner of the green house drinks coffee.
    ```prolog
        clue11(L) :- nationality(owner, H), color(green, H), drink(coffee, H).
    ```

12. The violinist drinks orange juice.
    ```prolog
        clue12(L) :- nationality(violinist, H), drink(orange juice, H).
    ```

13. The horse is in a house next to that of the diplomat.
    ```prolog
        clue13(L) :- pet(horse, H), nationality(diplomat, H2), neighbor(H, H2).
    ```

14. Milk is drunk in the middle house.
    ```prolog
        clue14(L) :- drink(milk, H), location(middle, H).
    ```

Optimizing Generate-Test

- It is more to generate fewer possibilities.
  ```prolog
  ok(X, Y, M) :- member(X, M), member(Y, M), X \= Y.
  ```

- Generates pairs where X = Y, then rejects them.
  ```prolog
  ok(X, Y, M) :- member(X, M, R), member(Y, R), X \= Y.
  ```

- Above, R is the "residue" of removing X from M.

- If M is known to contain no duplicates, the final check is not necessary.

Exercise: Extended member Predicate

- Extend member to have a 3rd argument: the residue left after the first element is removed from the list:
  ```prolog
  member([X | L], M, R) :- member(L, M, R), X \= head(L).
  ```

Generating with `append`

- Generates lists of solutions:
  ```prolog
  append([], L, L).
  ```

- For relational input:
  ```prolog
  relational
  append([], [1, 2, 3], [1, 2, 3]).
  ```

- For functional input:
  ```prolog
  functional
  append([1, 2, 3], [4, 5], Z) :-
  ```

- No solution found:
  ```prolog
  Z = [1, 2, 3, 4, 5],
  ```
  ```prolog
  no
  ```
Using a Generator as a “for” loop

?- for(I, 5, 8).
I = 5 ;
I = 6 ;
I = 7 ;
I = 8 ;
No

Definition:
for(M, M, N) :- M <= N.
for(I, M, N) :-
M < N,
M1 is M+1,
for(I, M1, N).

Caution: Won’t work in reverse, due to is.

Generating an Infinite Set

?- for(I, 5).
I = 5 ;
I = 6 ;
I = 7 ;
I = 8 ;
.
.
.

Definition:
for(M, M).
for(I, M) :-
M1 is M+1,
for(I, M1).

Caution: Won’t work in reverse, due to is.

Exercise: Generate all Pairs in N x N

?- pair(I, J).
I = 0 J = 0 ;
I = 0 J = 1 ;
I = 1 J = 0 ;
I = 0 J = 2 ;
I = 1 J = 1 ;
.
.
.

Non-deterministic Programming

- One interpretation of “non-deterministic”:
  - Find all solutions by finding one solution.
  - Solutions can here either be for the overall problem or a sub-problem.

Example of ND Programming

- permutation(X, Y) is true if list Y is a permutation of list X.
- An attempt:
  permutation(X, Y) :- sort(X, Z), sort(Y, Z).
- This is logical, but doesn’t work: the built-in sort is uni-directional.

Permutation

- permutation([], []).
- permutation([A | M]) :-
  member(A, L, Residue),
  permutation(Residue, M).
slowsort (joke)

% slowsort(X, Y) is true when Y is a sorted permutation of X.
% slowsort(X, Y) :- permutation(X, Y), sorted(Y).
sorted([A, B | X]) :- A @=< B, sorted([B | X]).
sorted([_])

N-Queens Problem:
ND Programming with Generate-Test Optimization

- Two queens on a chessboard are “attacking” if they are in a common row, column, or diagonal.
- Given a board size N, find a solution (or all solutions) for placing N queens so that no two are attacking.

Example, N = 4

Solution representation:
[[1, 3], [2, 1], [3, 4], [4, 2]]

Strategy

- An unoptimized generate-test would generate all possible boards, then test whether a board satisfies the constraints.
- Optimizations:
  - Generate: Generate only boards with a queen in each column.
  - Test: Add one column at time, reducing the test part to testing conflicts introduced by the newly-added column.

Solving Queens

Given a next column and a list of unoccupied rows:
- If rows is empty, succeed (all rows used).
- For the next unassigned column:
  - If there is a row where the queen is not being attacked, place it and recurse.
  - If no such row, fail (and backtrack).

Queens Top-Level

queens(N, Solution) :-
  range(1, N, Rows),
  queens(Rows, 1, Solution).

range(M, N, []) :- M > N.
range(M, N, [M | R]) :- M =< N, M1 is M+1, range(M1, N, R).
Queens Recursion

queens([], _, []). % basis

queens(Rows, Col, [[Col, Row] | Rest]) :-
    NextCol is 1 + Col,
    member(Row, Rows, RemainingRows),
    queens(RemainingRows, NextCol, Rest),
    nonAttacking(Rest, Col, Row).

Queens Non-Attacking

nonAttacking([], _, _).

nonAttacking([[Row1, Col1] | Pairs], Row, Col) :-
    Row1 - Row =\= Col1 - Col,
    Row - Row1 =\= Col1 - Col,
    nonAttacking(Pairs, Row, Col).

% This checks diagonals.
% Why don't we need to check row and column attacks?

The “42” Game
(homework problem)

● Given:
  ● A set of positive integers: [2, 4, 5, 7]
  ● A set of reusable operators: [+,-,*]
  ● A target: 24

● Construct an expression (as a syntax tree) showing how to make the target from the integers.

Approach to 42:
Non-Deterministic Programming

● If there is only one integer in the set:
  ● There is no choice. Either it is the same as the goal or not.

● Otherwise:
  ● Split the integers into two non-empty subsets.
  ● Compute a tree that could be constructed from each of the subsets.
  ● Choose an operator for the root joining the two trees.
  ● Check to see whether the overall tree meets the given target.

Splitting a List

● Only concerned with lists of 2 or more elements (why?)

?- split([1, 2, 3, 4], X, Y).
X = [1, 2, 3]
Y = [4] ;

X = [1, 4]
Y = [2, 3]

Splitting a List

● The tricky part is making sure that you get every way of splitting.

● Ideally you get back each way of splitting only once.
**Base Case**

- A list of exactly two elements is easy to split.

**How to split a list of > 2 elements**

- Remove an element E from the list (using an extended 'member' predicate).
- Recursively split the remaining elements into two.
- Add E back to the first list.
- Exploit symmetry by using an auxiliary predicate and calling it twice from the interface predicate:
  
  \[
  \text{split}(X, L, R) \leftarrow \text{split2}(X, L, R).
  \]
  
  \[
  \text{split}(X, L, R) \leftarrow \text{split2}(X, R, L).
  \]

**Strong recommendations**

- Don't evaluate a tree until the final tree is built.
- Don't try to optimize by evaluating sub-trees during the solution search (at least not for this assignment).

**Evaluating a Tree**

- Assume we are representing trees as lists:
  - A number is a tree
  - If T1 and T2 are trees, then \([\text{Op}, T1, T2]\) is a tree.

- Example:
  
  \[
  + 5 + \]
  
  \[
  6 - \]
  
  \[
  7 8
  \]

**A Tree Evaluator**

- \(\text{eval}(\text{Tree}, \text{Value})\).
- \(\text{eval}(N, N) \leftarrow \text{number}(N)\).
- \(\text{eval}([\text{Op}, T1, T2], N) \leftarrow ...\)

**Expressions as Lists Not Strictly Necessary**

- An alternate model (not used fall 2010) is to use the expressions themselves as trees.
- Expressions are not inherently evaluated in Prolog, so their parts can be recovered.
Prolog Uninterpreted Expressions

- Prolog has a built-in an infix operator precedence parser:
  - 3+4*5 is really:
    +((3, *(4, 5)))

  How can you be sure? Try unifying:

  ?- 3+4*5 = +(3, *(4, 5)).
  Yes

Evaluating an Expression

- The is operator will evaluate an expression, = (unification) will not:

  ?- X is 3+4*5.
  X = 23

  ?- X is +(3, *(4, 5)).
  X = 23

  ?- X = 3+4*5.
  No

  ?- X = +(3, *(4, 5)), Y is X.
  X = 3+4*5, Y = 23

Composing/Decomposing an Expression

- Infix operator =.. (called "univ") will build an expression from an operator and arguments, or take an expression apart:

  ?- X =.. [+ , 3, 4]. % compose
  X = 3+4

  ?- 3+4 =.. Y. % decompose
  Y = [+ , 3, 4]

Example for solve42 alternate

?- setof(Exp, solve42a([+, *, -], [2, 3, 4, 5], 24, Exp), Ans).

Ans =

[2*(3+(4+5)),
  2*(3+(5+4)),
  2*(4+(3+5)),
  2*(4+(5+3)),
  2*(5+(3+4)),
  2*(5+(4+3)),
  2*(3+4+5),
  2*(3+5+4),
  2*(4+3+5),
  2*(4+5+3),
  ...]

Quantifiers

- In addition to truth function operators of proposition logic, predicate logic introduces quantifiers for expressing variation over individuals:

  (\forall x) p(x) : for all x, p(x)

  [universal quantifier]

  (\exists x) p(x) : for some x, p(x)

  [existential quantifier]
Order of Quantifiers

- $(\forall x) (\exists y)$ knows($x$, $y$): Everyone knows someone.
- $(\exists x) (\forall y)$ knows($x$, $y$): Someone knows everyone.
- $(\exists x) (\forall y)$ $\neg$knows($x$, $y$): Someone knows no one.
- $(\exists x) (\exists y)$ knows($x$, $y$) $\land$ $x \neq y$: Someone knows someone other than him/herself.

Quantifiers in Prolog

- In most formulas, quantifiers are implicit:
  - If a variable appears in the head, it is for-all quantified in the rule.
  - If a variable appears in the body, but not the head, it is there-exists quantified.
- Examples:
  - $p(X, Y) :- q(X), r(X, Y)$ says: $(\forall x) (\exists y) (q(x) \land r(x, y)) \rightarrow p(x, y)$.
  - $p(X) :- q(X), r(X, Y)$ says: $(\forall x) (\exists y) (q(x) \land r(x, y)) \rightarrow p(x)$.

Quantifiers in Prolog

- The $\exists$ can be made explicit:
- Examples:
  - $p(X)$ :- $q(X), r(X, Y)$ says: $(\forall x) (\exists y) (q(x) \land r(x, y)) \rightarrow p(x)$.
  - $p(X)$ :- $Y ^{q(X), r(X, Y)}$ says the same thing.
  - $^$ is an "infix" version of $\exists$.

Where it Really Matters: setof

- Consider
  - setof($x$, $p(x, y)$, $z$).
- How is $Y$ quantified? If you want it to be $\exists$, the usual case, use:
  - setof($x$, $Y ^{p(x, y)}$, $z$).
- If you leave it off, it is a free variable, and may become bound in solving, in which case all other solutions would use the same $Y$.
- You won’t get all solutions for all $Y$ in this case.
- Typical use of the unquantified version:
  - $p(X, Z)$ :- setof($x$, $p(x, y)$, $z$).
  - Here there is a set of $Z$ for each possible $X$.

== in Prolog is not unification

- == is literal equality
- $a == a$ succeeds
- $a == b$ fails
- $X == a$ fails if $X$ is unbound (unlike =)
- $X = a, X == a$ succeeds ($X$ becomes bound)
- $X == Y$ fails if either is unbound

\== in Prolog is literal inequality

- $a \== a$ fails
- $a \== b$ succeeds
- $X \== Y$ succeeds if either is unbound
- There is no $\neq$ (not-unifiable) operator.
- Instead use $\neq x = y$ (it is not the case that $X = Y$).
Other comparison operators

- @<  compare arbitrary terms (e.g., lists)
- @>  in lexicographic order
- @=<
- @>=

Some Reversible Arithmetic can be Simulated with Lists

Number N is represented as a list of N 1’s

\[ \text{sum}([], Y, Y). \]
\[ \text{sum}(1|X), Y, (1|Z) :- \text{sum}(X, Y, Z). \]

The following doesn’t quite work for all inverses. A problem arises in factoring 0.

\[ \text{prod}([], Y, []). \]
\[ \text{prod}(1|X), Y, Z :- \text{prod}(X, Y, Z1), \text{sum}(Z1, Y, Z). \]

Example: Towers of Hanoi

Move only one disk at a time. Never place a larger disk on a smaller one.

Solving Towers of Hanoi

- Some approaches:
  - Pre-programmed solution
    - Recursive solution is easy in most languages
  - Let Prolog find solution using depth-first search
    - Trickier, but shows off Prolog’s capabilities
    - May not find shortest solution
  - Program breadth-first search in Prolog
    - Still trickier
  - Program iterative-deepening search
    - Easier than breadth-first

Pre-Programmed Towers of Hanoi (1)

- To move N disks from stack From to stack To:

Pre-Programmed Towers of Hanoi (2)

- To move N disks from stack From to stack To:
  - Move N-1 disks from stack From to stack Other (the stack other than From and To)

A key point throughout is that the N-1 disk moves can be done without violating the constraint that a larger disk not be put atop a smaller one.
Pre-Programmed Towers of Hanoi (3)

- To move N disks from stack \textit{From} to stack \textit{To}:
  - Move N-1 disks from stack \textit{From} to stack \textit{Other} (the stack other than \textit{From} and \textit{To})
  - Move 1 disk from stack \textit{From} to stack \textit{To}

Pre-Programmed Towers of Hanoi (4)

- To move N disks from stack \textit{From} to stack \textit{To}:
  - Move N-1 disks from stack \textit{From} to stack \textit{Other} (the stack other than \textit{From} and \textit{To})
  - Move 1 disk from stack \textit{From} to stack \textit{To}
  - Move N-1 disks from stack \textit{Other} to stack \textit{To}

Data Representation

- Number the disks 1, 2, 3, . . . smallest to largest.
- Use numeric value to detect size constraint.

Depth-First Towers of Hanoi (1)

Does not require a human to solve the puzzle. First characterize the possible moves.

This is a move from stack 1 to stack 2:

\[
\text{move}(\{1, 2\}, \{\{F1 \rightarrow R1\}, S2, S3\}, \{R1, \{F1 \rightarrow S2\}, S3\}) :-
\]

\[
\text{ok}(F1, S2).
\]

provided that it is ok to move disk F1 onto stack S2.
Depth-First Towers of Hanoi (2)

All the possible moves in six rules:

move([1, 2], [[F1 | R1], S2, S3], [R1, [F1 | S2], S3]) :- ok(F1, S2).
move([1, 3], [[F1 | R1], S2, S3], [R1, S2, [F1 | S3]]) :- ok(F1, S3).
move([2, 1], [S1, [F2 | R2], S3], [[F2 | S1], R2, S3]) :- ok(F2, S1).
move([2, 3], [S1, [F2 | R2], S3], [S1, R2, [F2 | S3]]) :- ok(F2, S3).
move([3, 1], [S1, S2, [F3 | R3]], [[F3 | S1], S2, R3]) :- ok(F3, S1).
move([3, 2], [S1, S2, [F3 | R3]], [S1, [F3 | S2], R3]) :- ok(F3, S3).

from / to          state before          state after          condition

Depth-First Towers of Hanoi (3)

When is it ok to move a disk onto a stack?
Assume the disks are represented by numbers 1, 2, 3, ...
with smaller numbers representing smaller disks.

ok(_, []). empty target stack
ok(A, [B | _]) :- smaller(A, B).
smaller(A, B) :- A < B.

Depth-First Towers of Hanoi (4)

towers([S1, S2, S3], Moves) will mean that Moves is a valid move sequence
that results in S1 and S2 being empty (so all disks are on S3).
towers([S1, S2, S3], Seen, Moves) means the same, except that Seen
will be a list of all previous states (to prevent infinite looping).

towers(InitialState, Moves) :- towers(InitialState, [], Moves).
towers([], [], [], []). N final state, no more moves

towers(InitialState, Moves) :-
  nonMember(InitialState, [ ], Moves).
move(Move, Before, After), only consider if Before not already seen
  towers(After, Before | Seen, Moves).

Depth-First Towers of Hanoi (5)

Auxiliary Predicates:

nonMember(X, L) :- \+ member(X, L).
member(X, [X | _]).
member(X, [_ | L]) :- member(X, L).

Exercise

Reverse the page by moving "forward" or jumping forward over a peg of either color.
Work out a depth-first solution in Prolog.
(You don't have to check for cycles, because there can't be any.)

Prolog Perspective

- A complete programming language
- Not a complete logic language
  - Restricted to "Horn Clauses"
  - Restricted form of negation
  - Quantifiers not completely general
  - Built in arithmetic not reversible
- More powerful logic systems exist, e.g.
  - Otter (see CS 80 or 151)
Contemporary Extensions of Prolog

- Constraint logic programming
- Inductive logic programming
- Lambda-prolog
- Goedel
- Parallel prologs
- Prolog++

... (The list is quite long.)