Assignment 1: Review and Warmup

Due: 1:15pm, Thursday, September 2 (!)

- The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
  - You contribute equally;
  - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
  - Your submission must authored solely by you, on a separate occasion.
- You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
- Bring a writeup/printout to class on Thursday. Illegible answers will get no credit. (For this reason, the grutors recommend LaTeX. But it's up to you.)
- Make sure your submission includes your name!

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1. Carefully diagnose the errors in the following two proofs:

   Theorem 1. *Horses have an infinite number of legs.*

   Proof. Horses have an even number of legs. Behind they have two legs, and in front they have fore legs. This makes six legs, which is certainly an odd number of legs for a horse. But the only number that is both odd and even is infinity. Therefore, horses have an infinite number of legs. □

   Theorem 2. *All horses are the same color.*

   Proof. We prove that any collection of horses is monochromatic by induction on the number of horses in the collection.
   Base case: Obviously, a set of one horse is a set of horses all with the same color.
   Inductive step: Assume we have a set of $k + 1$ horses.

   $$\overbrace{\text{horse}_1 \quad \text{horse}_2 \quad \cdots \quad \text{horse}_k}^{k \text{ of the same color}} \quad \text{horse}_{k+1}$$
   $$\text{horse}_1 \quad \underbrace{\text{horse}_2 \quad \cdots \quad \text{horse}_k \quad \text{horse}_{k+1}}_{k \text{ of the same color}}$$

   By the inductive hypothesis, the first k are the same color, and so are the last k. Thus, the entire set consists of $k + 1$ horses with the same color. □

2. You might know Euclid's proof that there an infinite number of primes. Here's an alternate proof due to Goldbach:

   Define the Fermat numbers

   $$F_n := 2^{2^n} + 1 \qquad \text{for } n = 0, 1, 2, \ldots$$

   There must be infinitely many primes because any two Fermat numbers are relatively prime. Why? Well, one can show that

   $$\prod_{k=0}^{n-1} F_k = F_n - 2 \qquad (n \geq 1) \tag{1}$$

   Thus, if m is a divisor of, say, F_k and F_n (with $k < n$), then m divides 2, and hence $m = 1$ or 2. But $m = 2$ is impossible since all Fermat numbers are obviously odd.

   Complete this proof by giving a careful inductive proof that Equation 1 holds for all $n \geq 1$.

3. Recall that

   $$\begin{array}{rcl} X \cup Y & := & \{\, z \mid z \in X \text{ or } z \in Y \,\} \\ X \cap Y & := & \{\, z \mid z \in X \text{ and } z \in Y \,\} \\ X \setminus Y & := & \{\, z \mid z \in X \text{ but } z \notin Y \,\} \\ X \subseteq Y & := & \text{every member of } X \text{ is a member of } Y \\ X = Y & := & X \subseteq Y \text{ and } Y \subseteq X \end{array}$$

   Carefully prove that

   $$A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$$

4. In Hofstadter's MIU-system (see the handout) you start out with one "axiom" and have 4 "rules of inference".

   - Show that MUIU is a "theorem" of the system (i.e., it can be produced by starting with your axiom and applying the rules of inference).
   - Prove that UM is *not* a theorem of the system (i.e., not derivable from the axiom by the given rules of inference).
   - Either show that MU is a theorem of the system, or prove that it is not. [This is a little tricky. If you can't get it, at least explain what approaches you've tried.]