

Assignment 2: Propositional Logic

Due: 1:15pm, Tuesday, September 7

- Emails about this assignment should be directed to cs81help@cs.hmc.edu.
 - The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
 - You contribute equally;
 - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
 - Your submission is authored solely by you, on a separate occasion.
 - You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
 - Bring a writeup/printout to class on Thursday. Illegible answers will get no credit. (For this reason, the grutors recommend \LaTeX . But it's up to you.)
 - Make sure your submission includes your name!
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Reading

In Huth & Ryan, read Chapter 1 *up to and including* Section 1.4.1 (except for Section 1.3, which you are free to skip).

One comment on Section 1.4.1: on Tuesday we will talk more about the relationship between provability (theorems) and truth (valuations) in propositional logic.

In particular, if there is a valid proof from assumptions to a conclusion, then every valuation that makes the assumptions true also makes the conclusion true. (As a special case, when a result can be proved without any assumptions, then every valuation will make this result true).

It follows, then, that if there is a valuation that makes the assumptions (if any) true and the conclusion false, then there is no proof of this conclusion from these assumptions.

Exercises

1. Carefully explain what is wrong with each of following proofs, which purport to prove p from no assumptions (except for temporary assumptions in sub-proofs).

<p>1 p Assumption 2 p Copy 1 3 $p \rightarrow p$ \rightarrow_i 1, 2 4 p \rightarrow_e 3, 2</p>	<p>1 $p \vee \neg p$ LEM 2 p Assumption 3 p Copy 2 4 $\neg p$ Assumption 5 \perp \neg_e 2, 4 6 p \perp_e 5 7 p \vee_e 1, 2–3, 4–6</p>
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2. Give proofs for the following propositions from Exercise 1.2 (pages 79–80) of Huth & Ryan: 1(k), 1(p), 1(s), 3(c), 3(e), 3(l).

3. **Pierce’s Law** is the formula

$$((p \rightarrow q) \rightarrow p) \rightarrow p$$

- (a) By using truth tables, show that Pierce’s law is a tautology, (i.e., that

$$\models ((p \rightarrow q) \rightarrow p) \rightarrow p,$$

or equivalently that this formula is true under every valuation).

- (b) Prove Pierce’s Law.

Hint: As mentioned in class (and in Section 1.2.5 of Huth & Ryan), constructive propositional logic rejects the use of “indirect” arguments. Consequently, in constructive logic one cannot use the excluded middle ($\vdash p \vee \neg p$), double-negation elimination ($\neg\neg p \vdash p$), and proof by contradiction (if $\neg p \vdash \perp$, then $\vdash p$) for an arbitrary proposition p . In constructive logic, Pierce’s Law is not provable.

4. **OS Wars:** Consider the following argument:

- Assumption 1: If my computer runs Windows, then Microsoft got my money.
- Assumption 2: If my computer runs Mac OS X, then Apple got my money.
- Conclusion: At least one of the following statements is true:
 - If my computer runs Mac OS X, then Microsoft got my money
 - If my computer runs Windows, then Apple got my money

Either provide a formal proof that this argument holds, or find a valuation where the assumptions are true but the conclusion is false. You may want to abbreviate certain propositions (e.g., as p , q , etc.) for the purposes of your valuation or your proof, but be sure to define any such abbreviations.

5. Answer the following survey questions:

- (a) How long did you spend on this assignment?
- (b) What was easiest? What was most difficult?
- (c) With whom, if anyone, did you collaborate with on this assignment?

Further Hints

- Make sure you understand all the sample proofs in Section 1.2!
- A common question is “What do I do with an *assumption* of the form $\neg A$?” The answer is “It depends.”

However, the most common pattern is to show that other assumptions (either those you were given or those you have assumed inside a box) imply A , and then use the fact that you’ve reached a contradiction.

[See lines 3–5 in the left half of Example 1.20, or lines 2–4 in the second proof on page 25.]

- Another common question is “How do I prove a *conclusion* of the form $A \vee B$?” The answer is “It depends.”

However, unless you can show that A is definitely true or that B is definitely true, the most common pattern is to use proof by cases (a.k.a. or-elimination). In one case prove A (and hence $A \vee B$ by or-introduction), and the other case prove B (hence $A \vee B$). In either case, $A \vee B$, so we can conclude it must be true.

Which two cases? Well, if you already have some other disjunction around (assumed or proved) you might do cases on which half is true. Alternatively, you can use the law of the excluded middle to prove your own disjunction $C \vee \neg C$ (for some specific and relevant formula C); then do one case where C is assumed, and one case where $\neg C$ is assumed.

[See Example 1.24.]