

Assignment 8: Regular and Nonregular Languages

Due: 1:15pm, Tuesday, November 2

- Emails about this assignment should be directed to `cs81help@cs.hmc.edu`.
 - Grutor office hours in Platt are **Thursdays 7–9pm**, Sundays 8–10pm and Mondays 9–11pm; Prof. Stone’s office hours in Olin 1251 are MW 4–5pm and TR 3–4pm and by appointment.
 - The usual collaboration rules apply. You may *discuss* an exercise with any other student(s) currently taking CS 81 as long as:
 - You contribute equally;
 - You come away from this discussion only with *understanding in your head* — no written materials or computer notes may be retained;
 - Your submission is authored solely by you, on a separate occasion.
 - You should refer only to materials from this semester of CS 81 (lecture notes, handouts, textbooks, grutors, profs, etc.).
 - Bring a writeup/printout to class on the due date. Illegible answers will get no credit.
 - Make sure your submission includes your name!
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1. Read pp. 77–82 and pp. 99–109 of Sipser. Come up with (at least) one question about the reading where you’re not sure of the answer. These may relate to points where the book is confusing, or simply to some related question or conjecture that occurs to you while doing the reading.
2. Do Exercise 1.4(g) on page 83.
3. Provide clear and convincing proofs for Problem 1.42, page 89. (The notation in the book is a little confusing: $a_1 \cdots a_k$ is a *single* word in A , and the a_i ’s are pieces of that word. Also, any of the a_i ’s might be the empty string. So for example, strings in the shuffle of a^+ and b^+ include $abab$, $aabbaabb$, $aabbaa$, ba , $babbabbaaa$, and so on. The intuition is to take a single string from A and a single string from B and do the equivalent of one “riffle shuffle” of two piles of cards.)

4. Define

$$\text{Prefix}(L) := \{ w \in \Sigma^* \mid \exists z \in \Sigma^*. wz \in L \}.$$

Intuitively, w is in $\text{Prefix}(L)$ if and only if it is a prefix of some word in L . Prove that if L is a regular language then $\text{Prefix}(L)$ is also regular.

5. Do Problem 1.32, page 88

6. Do Problem 1.33, page 89

7. Do Problem 1.34, page 89

8. Do Problem 1.53, page 91.

9. Here are a few languages with $\Sigma = \{0, 1\}$. Decide whether each is regular or not, and prove your answer is correct.

(a) The set of all strings x having some non-empty substring of the form www , e.g., 111 and 00101011 . (You may make use of the following true fact: There are arbitrarily long strings in Σ^* that *do not* contain any non-empty substring of the form www .)

(b) The set of strings having the property that in every prefix, the the number of 0's and the number of 1's differ by no more than 2.

(c) The set of strings s for which there exists an integer $k > 1$ (possibly depending on s) such that the number of 0's in s and the number of 1's in s are both divisible by k .

10. Do Exercise 2.4(a-f), page 128.