CS 81: Logic and Computability

August 31, 2010

Fill out the survey and
then skim the syllabus
while I'm taking pictures...

CS 81:
Logic and Computability

AKA

Introduction to Formal Systems

Proofs vs. Computation

<u>Defining the System</u>

Well-Formed Formulas	Legal Programs
Axioms/Inference Rules	Language Definition

<u>Specific Instances</u>

Assumptions	Initial Conditions
Proof Step 1	Computation Step 1
Proof Step 2	Computation Step 2
...	...

A Selective History

Aristotle

Boole

Frege

Russell

Hilbert

Gödel

Church/Turing

The Need to Study Logic

This dog is a father

This dog is mine

Therefore, this dog is my father

The Need to Study Logic

Prof. Ran is Prof. Libeskind-Hadas
This is Prof. Libeskind-Hadas

Therefore, this is Prof. Ran

Man is animal
Fluffy is animal

Therefore, Fluffy is Man

The Need to Study Logic

The Steelworkers were unionized

Therefore, they had no missing electrons

Ambiguous Language

John saw a picture
of the prettiest girl he had ever seen
hanging on a locker door.

Questionable Logic

Put $U = \{ S \mid S \text{ is a set} \}$

An Inductively Defined Set

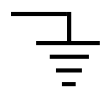
I'm thinking of a set S.

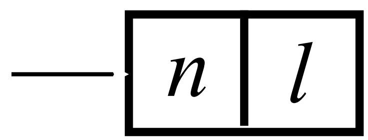
0 is in S.

If n is in S, then n' is in S.

S is the smallest set obeying these rules.
(Equivalently, every element of S can be shown to belong to S by using these rules finitely many times.)

An Inductively Defined Set

 is in L.

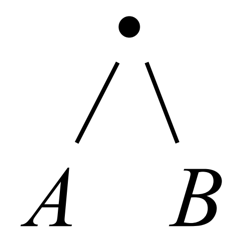
If n an integer and l is in L,
then  is in L.

L is the <u>smallest</u> set obeying these rules.
(Equivalently, every element of L can be shown to belong to L by using these rules <u>finitely</u> many times.)

An Inductively Defined Set

• is in T.

If A and B are in T, then so is



T is the smallest set obeying these rules.
(Equivalently, every element of T can be shown to belong to T by using these rules finitely many times.)

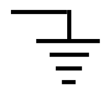
Structural Induction

If the *construction of* an inductively defined set guarantee/preserve some property

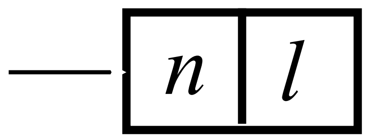
then

every element of the set has that property.

Lists in *L* **are finite**

 in L is finite? ✔

If n an integer and l in L **is finite**.

then  in L **is finite**? ✔

QED

Propositional Logic

Proposition / Formula

An statement that could
be argued as true or false

Do You Recognize These Symbols?

$$(\text{CS } 60 \lor \text{CS } 42 \lor \text{CS } 52)$$

$$\land$$

$$(\neg \text{Math } 55 \rightarrow \text{CS } 55)$$

$$\land$$

$$\neg \text{CS } 81.$$

Well-Formed Formulas

(an inductive definition!)

p, q, r, etc., are WFFs.

$\top$ and $\bot$ are WFFs.

If A is a WFF then so is $\neg$A.

If A and B are WFFs then so are
(A$\wedge$B), (A$\vee$B), (A$\rightarrow$B), and (A$\leftrightarrow$B).

Simplifications

We will sometimes omit parentheses:

1. Binding tightest to loosest: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

2. Left-associative: $\wedge$, $\vee$, $\leftrightarrow$
   Right-associative: $\rightarrow$

$p \wedge q \rightarrow r$

$p \rightarrow q \rightarrow p$

$\neg p \wedge r \vee p \wedge \neg q$

$(p \rightarrow q \rightarrow r) \rightarrow (p \rightarrow q) \rightarrow p \rightarrow r$

Exercise

Choose specific propositions for p and q, and express these formulas in English.

$p \wedge q \rightarrow q \wedge p$

$p \rightarrow p$

$p \vee \neg p$

$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$\neg\neg(p \vee \neg p)$

Truth vs. Provability

A is true:

$$\models A$$

A is provable:

$$\vdash A$$

How should these relate?

Which is easier to demonstrate?

Theorems

(An inductively defined Set!)

Axioms/Axiom Schemes:

... is a theorem.

... is a theorem.

Rules of Inference:

If ... is a theorem and ... is a theorem
then ... is also a theorem.

Example: A Hilbert-Style System

Axiom Schemes (For any formulas F, G, and H):

$$\vdash F \rightarrow (G \rightarrow H)$$

$$\vdash (F \rightarrow G \rightarrow H) \rightarrow (F \rightarrow G) \rightarrow F \rightarrow H$$

$$\vdash (\neg G \rightarrow \neg F) \rightarrow (F \rightarrow G)$$

Rule of Inference: (For any formulas F and G):

If $\vdash F$ and $\vdash F \rightarrow G$ then $\vdash G$.

Theorems

(An inductively defined Set!)

Axioms/Axiom Schemes:

... is a theorem.

... is a theorem.

} true?

Rules of Inference:

If ... is a theorem and ... is a theorem
then ... is also a theorem.

} truth-preserving?