Jewels of Computer Science

CS 81: Computability and Logic

October 14, 2010

Outline

- Languages vs. State Machines (Recognizers)
  - Every state machine specifies a language
  - For every language, there are state machines that specify it.
  - For every language, there is a minimal state machine
- What languages have finite state machines?

Minsky on Machines (1967)

When the term “machine” is used in ordinary discourse, it tends to evoke an unattractive picture. It brings to mind a big, heavy, complicated object which is noisy, greasy, and metallic; performs jerky repetitive, and monotonous motions; and has sharp edges that may hurt one if he does not maintain sufficient distance...

Abstract Machines

- Abstraction of computation.
- Simplest form: Recognizers

abbaabb...

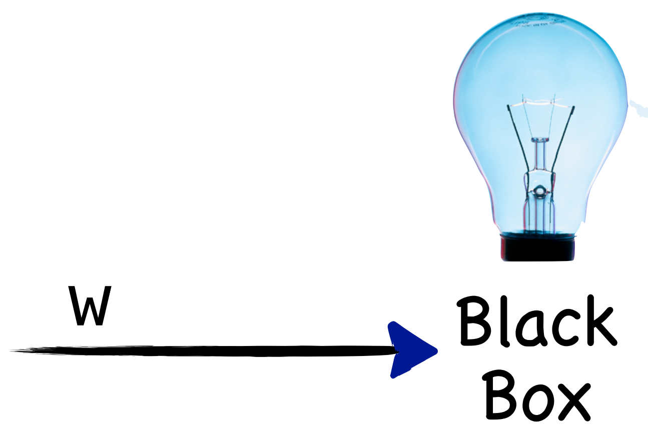
Black Box

(Light goes on or off after each symbol)

Language of a Machine

"w is accepted" : the light is on when w is fully entered

"w is rejected" : the light is off when w is fully entered

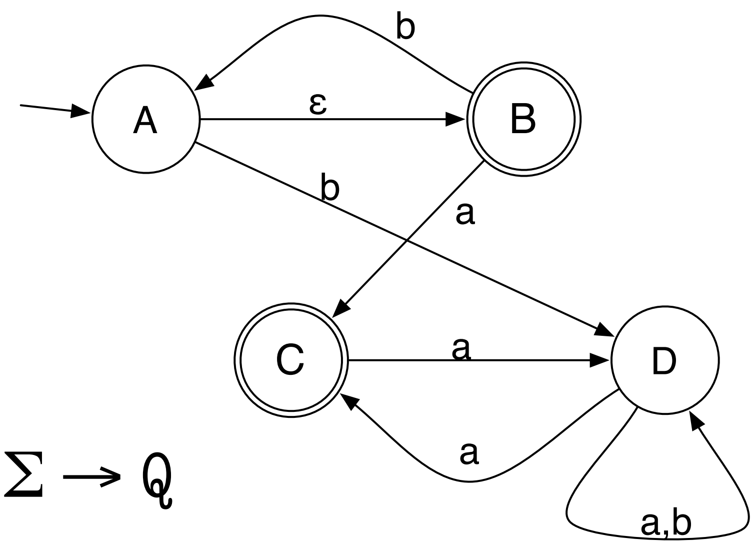


Every machine corresponds to a language (accepted strings)

Inside the Black Box

Mathematically, a <u>state machine</u> consists of:

1. an alphabet Σ
2. a collection of states Q;
3. a transition relation $\rightarrow \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times Q$
   where $q \xrightarrow{\sigma} q'$ means that (q, σ, q') is in the relation;
4. one initial state $q_0 \in Q$;
5. a set of final/accepting states $F \subseteq Q$.



<u>finite</u>: Q is finite

<u>deterministic</u>: transition function $\delta: Q \times \Sigma \rightarrow Q$

Terminology

Finite State Machines also called Finite Automata

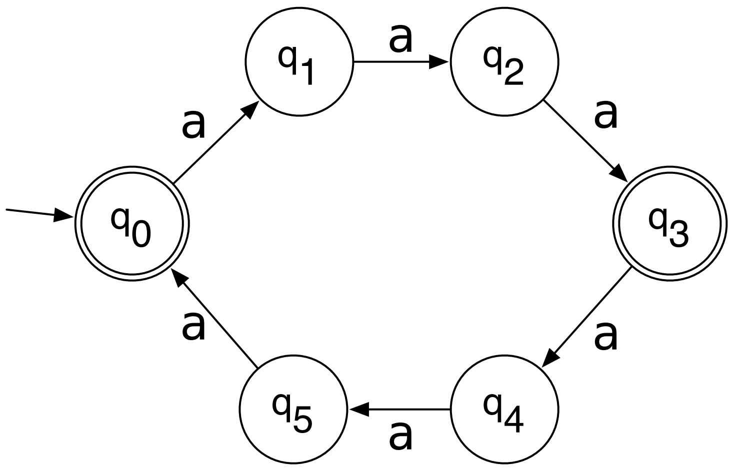
Hence, you may see references to

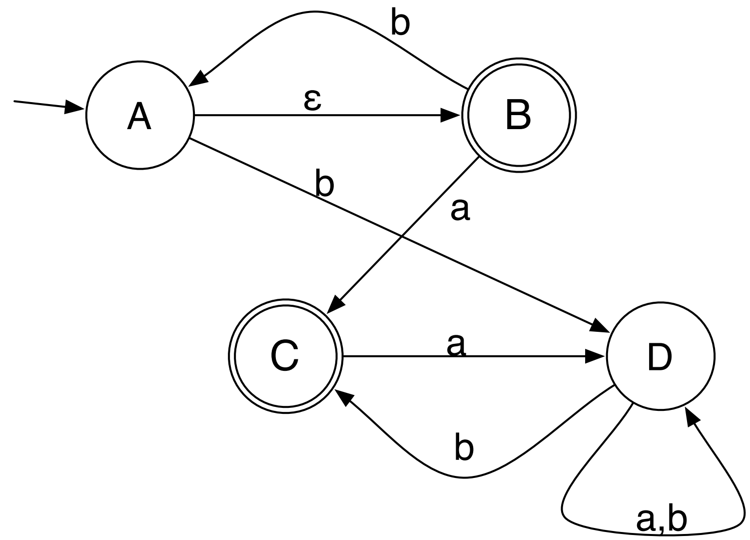
- NFSM vs. DFSM
- NFA vs. DFA

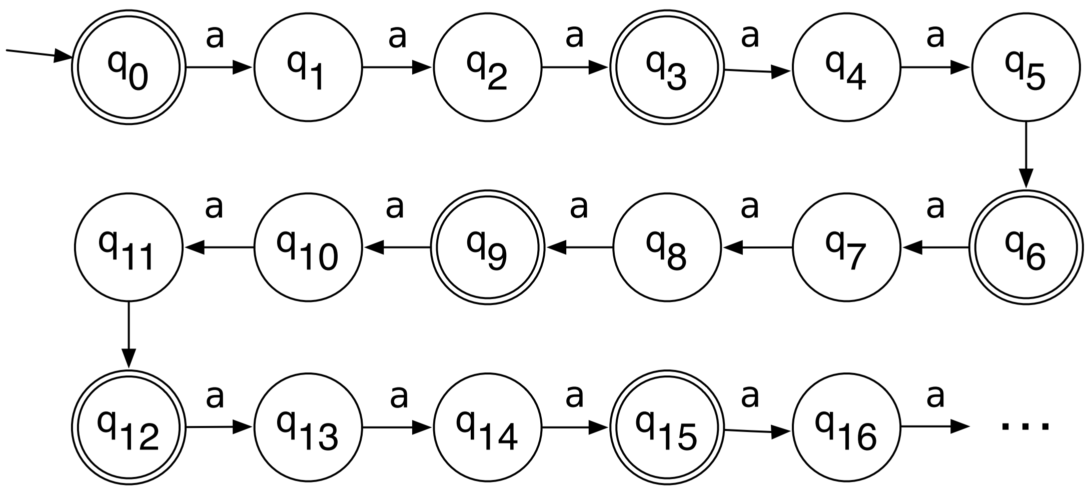
Machine Behavior

- A machine starts in state q_0.
- From a current state q it can change state to q' with input σ provided that $q \xrightarrow{\sigma} q'$
- From a current state q it can change state to q' spontaneously provided that $q \xrightarrow{\varepsilon} q'$
- The machine accepts a string $w \in \Sigma^*$ provided there is some path, spelling out w, that starts at q_0 and ends at <u>some</u> $q \in F$.

What's Accepted?





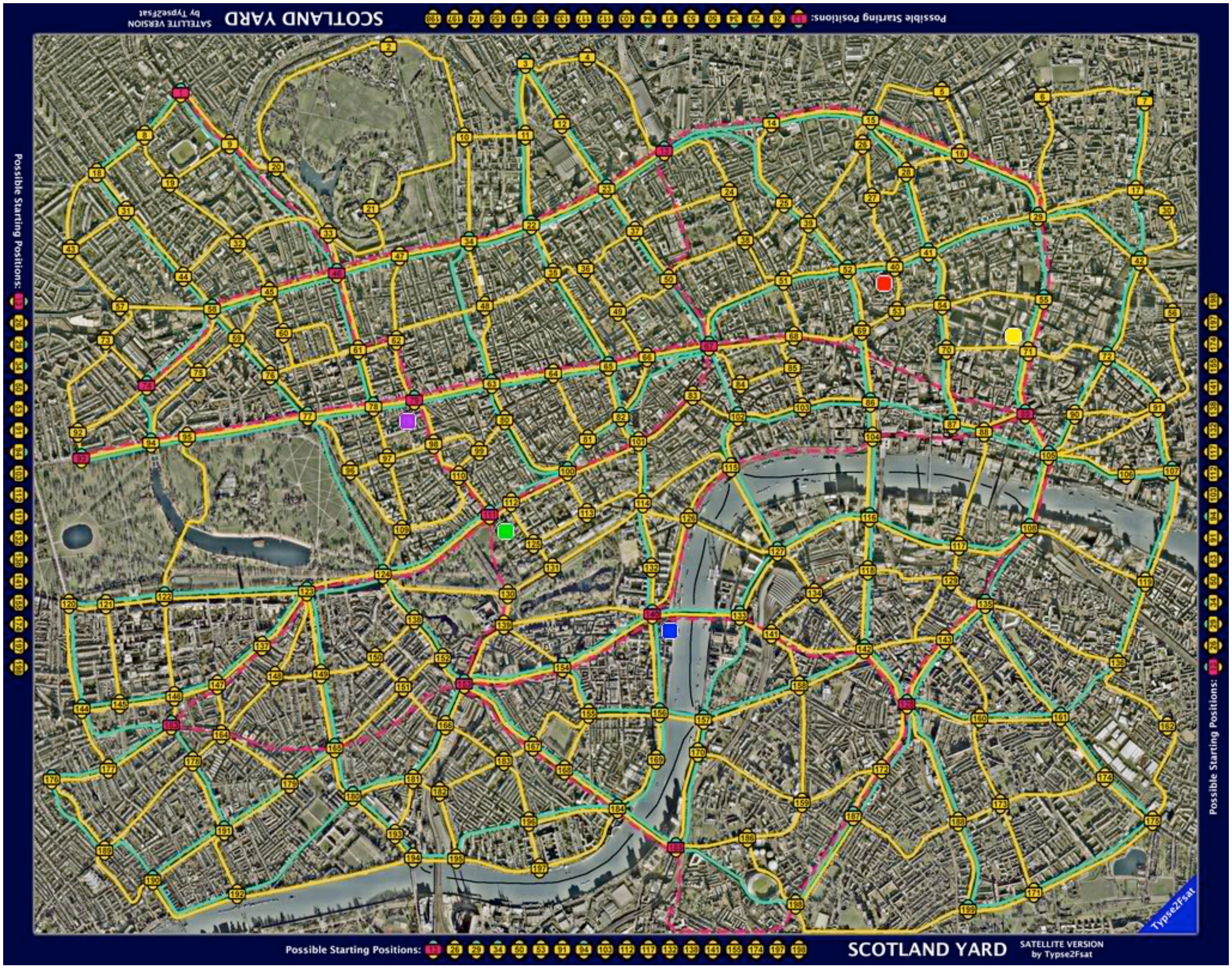


Regular Languages

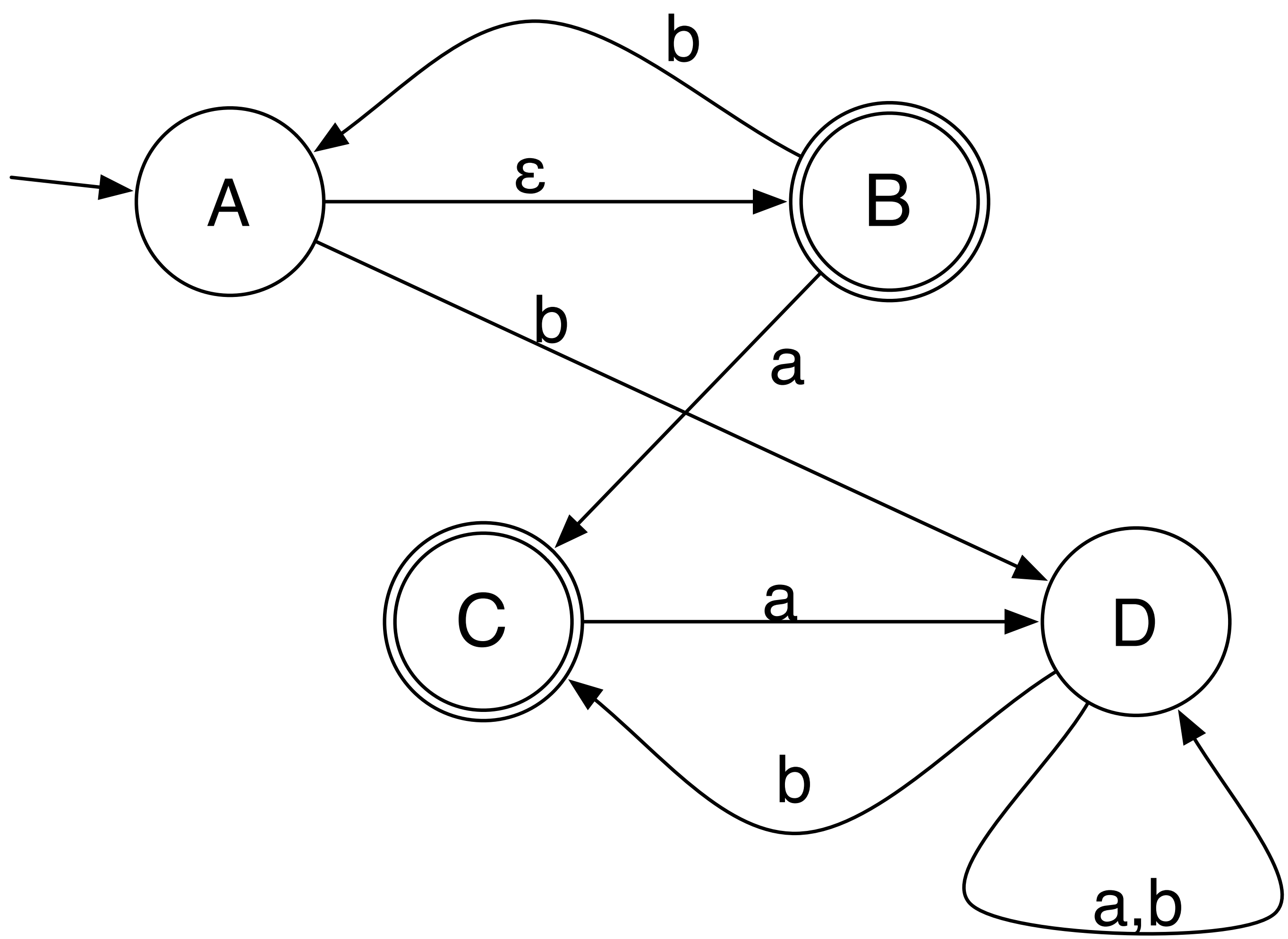
The following are equivalent:

- There is an NFA accepting the language L
- [Rabin and Scott]: There is a DFA accepting L
- [Kleene] L is regular.
- [Myhill-Nerode] $\{ L_w \mid w \in \Sigma^* \}$ is finite.

Digression: “Scotland Yard” (the game)



From NFA to DFA: the **Subset Construction**



Regular Languages

(an inductively defined collection of sets!)

- $\varnothing$ is regular
- $\{a\}$ is regular for every $a \in \Sigma$.
- If L is a regular language, then so is L^*
- If L and M are regular, then so are LM and $L \cup M$

Regular Expressions

(an inductive definition!)

- $\emptyset$ is a regexp
- a is a regexp for every $a \in \Sigma$
- If r is a regexp, so is (r^*)
- If r_1 and r_2 are regexps, so is $(r_1 r_2)$ and $(r_1 | r_2)$

Convention: $ab^*|c^* = (a(b^*)) \mid (c^*)$

Regular Expressions

- Regular expressions abbreviate regular languages

$L(\emptyset) = \emptyset$

$L(\varepsilon) = \{\varepsilon\}$

$L(a) = \{a\}$

$L(r^*) = (L(r))^*$

$L(r_1 r_2) = L(r_1)\, L(r_2)$

$L(r_1 | r_2) = L(r_1) \cup L(r_2)$

Defined by induction/ recursion on the regular expression!

- $L(r_1) = L(r_2) \nrightarrow r_1 = r_2$!
- We say that "w <u>matches</u> r" if $w \in L(r)$.

Examples

0 | 1

(0|1)*

(0|1) 0* 1*

0*110* | 1*001*

Regexp Exercise

$\Sigma = \{0,1\}$

- Strings where every 1 is followed by a 0.
- Strings where no 1 is followed by a 0.
- Strings where every 1 is preceded by and followed by a 0.

From Regexp to NFA

(by induction/recursion on the regexp)

- ✔ $\emptyset$ is a regexp
- ✔ a is a regexp for every $a \in \Sigma$
- ✔ If r is a regexp, so is (r^*)
- ✔ If r_1 and r_2 are regexps, so is $(r_1 r_2)$ and $(r_1 | r_2)$